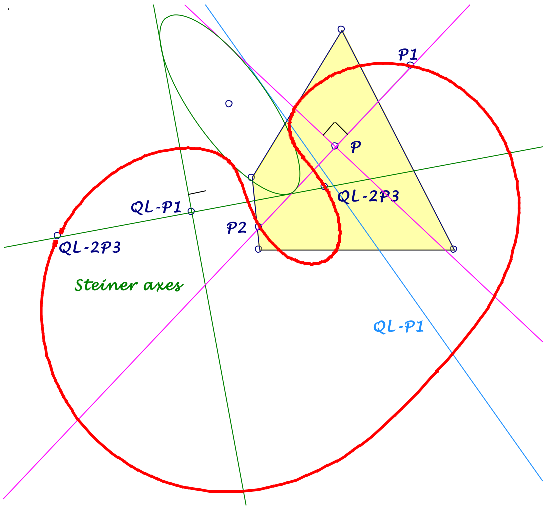
EQF-Note 2015-11-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Quartics

Here is a generalization of the quartic in QFG-message 1354. These generalized quartics are the loci of CSC-partners on tangents of special conics, depending on a point P. They are CSC-invariant and anallagmatic.

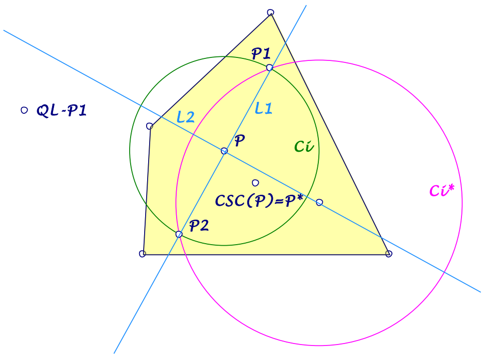


Construction of a P-Quartic

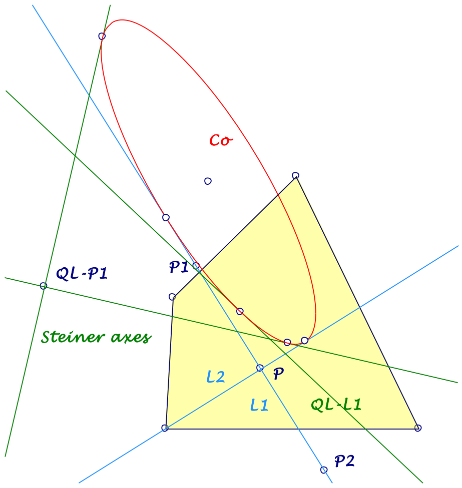
Let *P* be the defining point and *P** its image wrt *QL-Tf1*, shortened *CSC*. Take a circle *Ci* round *P* with radius

$$\sqrt{P.P^{*} \times P.QL-P1}$$

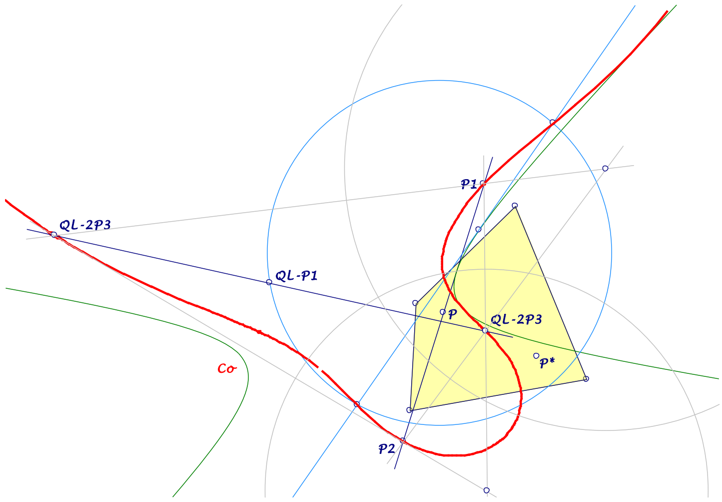
and the intersections P_1, P_2 with its *CSC*-circle *Ci** with the lines $L_1 = P_1P_2$ and L_2 perpendicular bisector of P_1P_2.



Let *Co* be the defining conic tangent to …
… the Newton line *QL-L1*,
… the Steiner axes
… and the lines L_1 and L_2.



The *CSC*-partners on tangents at this conic give the *P*-quartic.



Properties:

- **The *P*-quartic contains …**
  … the *CSC*-fixed points *QL-2P3*,
  … the points P_1, P_2 (see above).

- **The *P*-quartic is *CSC*-invariant (evident).**

- **The *P*-quartic is invariant wrt a combination of …**
  … an inversion wrt the circle *Ci* (see above)
  … and a reflection in the angle bisector of *QL-P1.P.P.**

- **The *P*-quartic is invariant wrt a combination of …**
  … an inversion wrt a circle round *P with radius**

  $$\sqrt{P^*.P \times P^*.QL-P1}$$

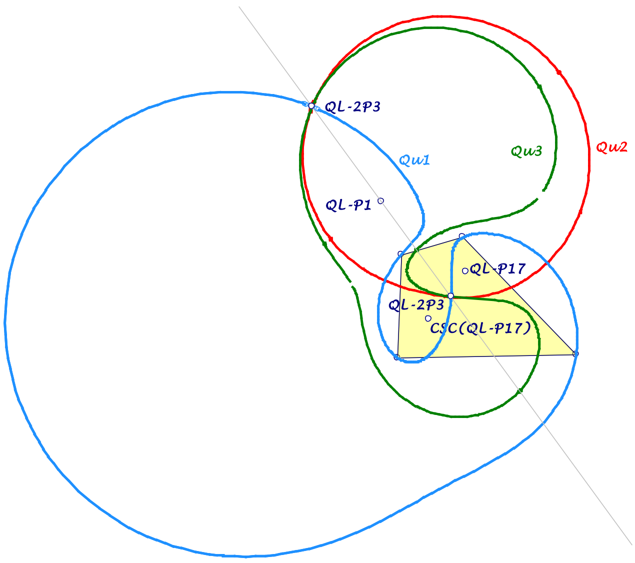
  … and a reflection in the angle bisector of *QL-P1.P*.P*.

- **The *P*-quartic is anallagmatic, centers for the inversion circles are *P_1.QL-2P3a* ∩ *P_2.QL-2P3b* and *P_2.QL-2P3a* ∩ *P_1.QL-2P3b*.**

Final remarks

Qu1: This quartic – **not a *P*-quartic** – is described in *QFG*-message 364. Its conic is tangent to the *QL*-diagonals, the Steiner axes and *QL-L2* (with orthogonal tangents to *QL-P17*).
Qu2: For *P* = *QL-P17* the *P*-quartic is described in *QFG*-message 1354.
Qu3: For *P* = *CSC(17)* the *P*-quartic is described in *QFG*-message 1363.

The quartics *Qu2* and *Qu3* are tangent in *QL-2P3*, they have orthogonal intersections with *Qu1*.
Bernard Keizer considers a related quadrilateral *DQL* wrt the diagonals and *QL-L1* with the corresponding *CSC*-transformation *CSdiagC* (see *EQF*-message *1355*). He found, that *Qu1* and *Qu2* are not only *CSC*-invariant wrt the reference *QL*, they are also *CSdiagC*-invariant. **This holds also for *Qu3***.



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