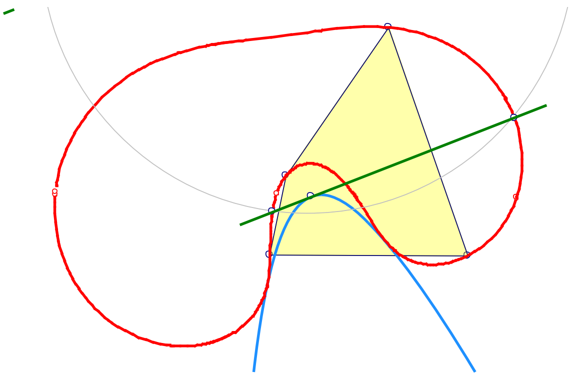
EQF-Note 2015-12-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A Third Quartic for a Quadrigon

In QFG-messages 362 and 363 two circumquartics of a quadrigon are described. Here is the geometry of a third quartic, related to a conic wrt CSC-partners on its tangents. This quartic is anallagmatic and invariant wrt several transformations.



Preliminary

The *CSC*-partners on tangents of a given conic give a quartic:

- For a *QL*-conic, tangent …
  … to the Steiner axes,
  the quartic is unipartite.

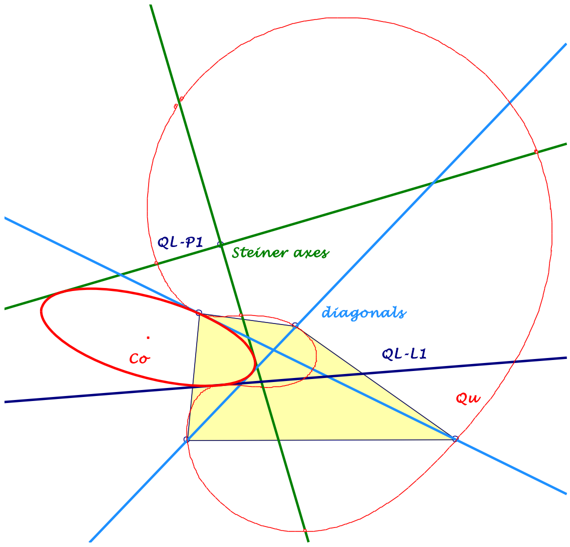
- For a *QL*-conic, tangent …
  … to the Steiner axes
  … and the Newton line *QL-L1*,
  the quartic is a *P*-quartic (see #1365 attached).

- For a *QG*-conic, tangent …
  … to the Steiner axes
  … and the diagonals,
  the quartic is a *QG*-circumquartic.

- **For a *QG*-conic, tangent …**
  … to the Steiner axes
  … and the diagonals
  … and the Newton line *QL-L1*,
  there is a special *QG*-circumquartic.

Here only the last quartic – shortened ***Qu*** − shall be treated, related to the special conic − shortened ***Co***.

The Quartic *Qu*



- The quartic *Qu* is an unipartite circumquartic of the quadrigon.

- The quartic *Qu* contains …
  … the *CSC*-fixed points *QL-2P3*
  … and the points *QL-2P2* on the cubic *QL-Cu1*.

- If there are conic-points, whose *CSC*-partner lies on its tangent, the conic *Co* contacts the quartic *Qu* in these points.

- The quartic *Qu* is invariant wrt the *CSC*-transformation *X* ⟶ *X** (evident).

Beside the *CSC*-transformation *X* ⟶ *X**, there is a *QG*-transformation *X* ⟶ *X^*, described in #362: The image *X^* is the second intersection of the two circles through *X* and opposite *QG*-vertices.

- The quartic *Qu* is invariant wrt the *QG*-transformation *X* ⟶ *X^*.

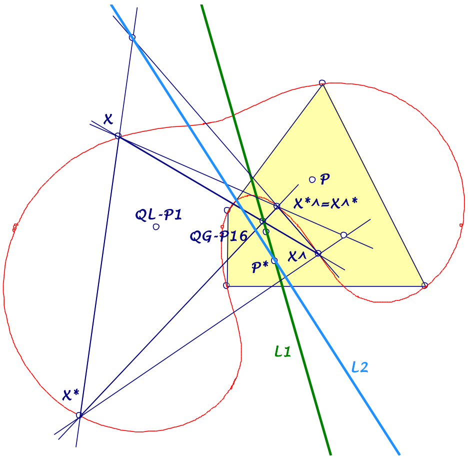
The Image Points, *X, *X^*, *X*^* = *X^****

The geometry of the four points *X*, *X**, *X^*, *X*^* = *X^** is worth to be mentioned:

- For points *X* on the quartic *Qu* …
  … *XX^* and *X*X*^* intersect on a line L_1 through *QG-P16* of the reference quadrigon,
  … *XX** and *X^X^** intersect on a second line L_2.

1. Consider the quadrigons X.X^.X*.X*^:

- These quadrigons have the same Miquel point *QL-P1* as the reference quadrigon.
- The points *QG-P16* of these quadrigons lie on the line L_1 collinear with *QG-P16* of the reference quadrigon.



2. Consider the quadrigons X.X*.X^.X^*:

- These quadrigons have the same Miquel point in
$$P = CSC(L_1 \cap L_2).$$
- The points *QG-P16* of these quadrigons lie on the line L_2.

3. Consider the quadrigons X.X*.X*^.X^:

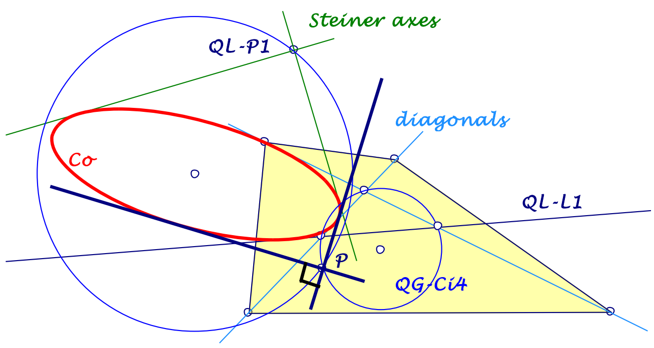
- These quadrigons have the same Miquel point in
$$P^* = L_1 \cap L_2 .$$
- The points *QG-P16* of these quadrigons lie on a circle, which is the *CSC*-image of the line L_2.

The Point $P = CSC(L_1 \cap L_2)$

The point *P* has a special geometry wrt the defining conic *Co* and the quartic *Qu*:

- *P* is a point on the quadrigon circle *QG-Ci4*.

This circle contains the diagonal crosspoint *QG-P1* and the diagonal midpoints on the Newton line *QL-L1*.



- *P* is a point on the circle round the center of the conic *Co* through the Miquel point *QL-P1*.

This circle is the locus of points with orthogonal tangents to the conic *Co*. The 2nd intersection with *QG-Ci4* is a Plücker point for the quadrilateral of the Steiner axes and the diagonals.

- The quartic is invariant wrt a *CSC*-analog transformation $X \rightarrow X´$, centered in *P*, which swaps *P** and *QL-P1*.

- The quartic *Qu* contains the fixed points of $X \rightarrow X´$.

- For points on the quartic *Qu* the transformation $X \rightarrow X´$ is the same as $X \rightarrow X$^.

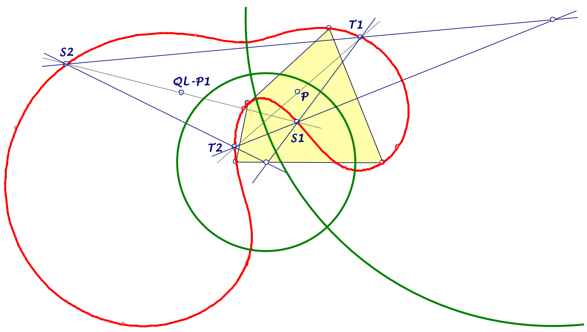
- The quartic is invariant wrt a *CSC*-analog transformation $X \rightarrow X´´$, centered in *P**, which swaps *P* and *QL-P1*.

- For points on the quartic *Qu* the transformation $X \rightarrow X´´$ is the same as $X \rightarrow X$^*.

The transformations $X \rightarrow X^*$, $X \rightarrow X´$, $X \rightarrow X´´$ are the *CSC*-analog transformations of the triangle *QL-P1.P.P**, centered in a vertex swapping the other two vertices.

- The quartic *Qu* is anallagmatic.

Let S_i be the *CSC*-fixed points *QL-2P3* and T_i the fixed points of $X \rightarrow X´$, then the centers of inversion are $S_1T_1 \cap S_2T_2$ and $S_1T_2 \cap S_2T_1$.



- The quartic is a *P*-quartic (see #1365 attached).

In this way the defining conic *Co* is tangent to the Newton line of the reference quadrigon and inscribed the cyclic quadrigon of the axes of $X \rightarrow X^*$ and $X \rightarrow X´$.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de