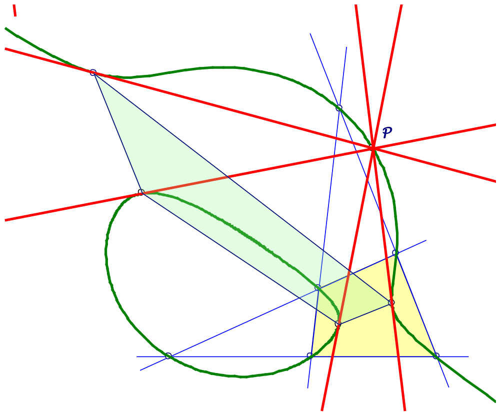
EQF-Note 2015-12-04

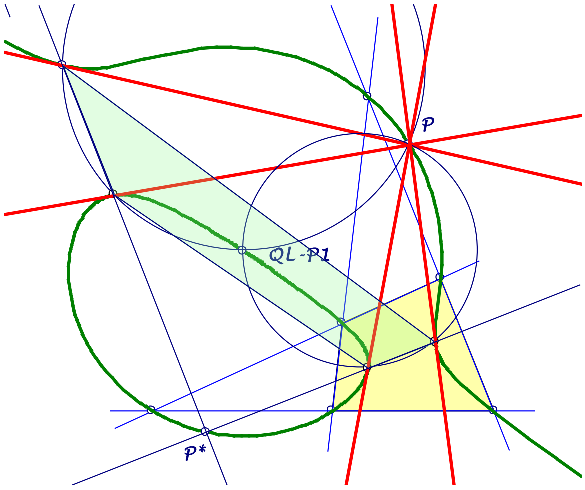
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Cu1 as QA-Cu1

What about the four possible tangents from a point on the cubic QL-Cu1 at QL-Cu1? This leads to a quadrangle of contact points (not always real), whose cubic QA-Cu1 is QL-Cu1 of the reference quadrilateral.



The central curve for a quadrilateral *QL* is the *QL*-Quasi Isogonal Cubic *QL-Cu1*, invariant wrt the transformation *QL-Tf1* – shortened *CSC*. In a first step a construction is given for the tangents from a point *P* on *QL-Cu1* at *QL-Cu1*:
… Let *P** be the *CSC*-image of *P*.
… Let L_1 and L_2 be the orthogonal angle bisectors in *P** wrt two opposite points of the quadrilateral.
… The intersections of L_i with their *CSC*-circle *CSC*(L_i) give four points (not always real).
… These four points are the contact points of the tangents from *P* at *QL-Cu1*.



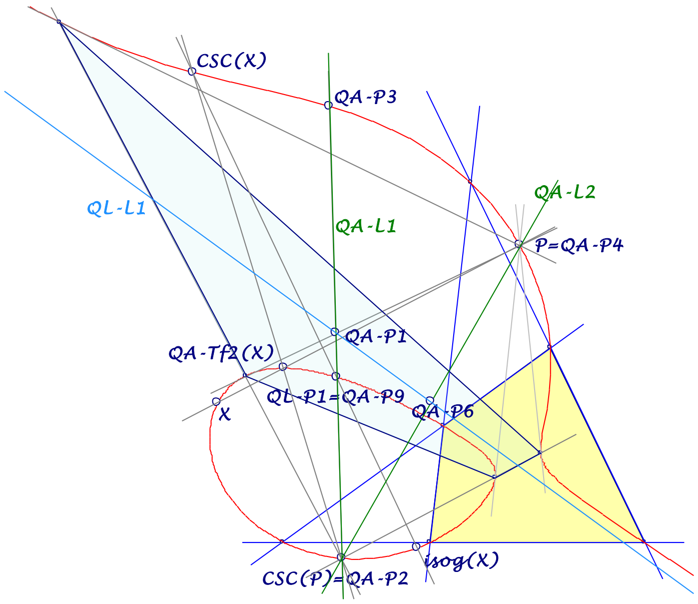
In a second step we study the quadrangle of the four contact points depending on a chosen point *P* on *QL-Cu1*. **It is a special quadrangle, for the vertices lie on two orthogonal lines.**
Nomination: The first letters *QA* refer to such a quadrangle, *QL* to the reference quadrilateral.
The main result:

- **The Cubic *QA-Cu1* is the Cubic *QL-Cu1*.**

- One vertex of the diagonal triangle *QA-Tr1* is *CSC(P)*.

- The vertices of the diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2* lie on *QL-Cu1*.

- *QL-Cu1* is invariant wrt *QA-Tf2* and isogonal invariant wrt the Miquel triangle *QA-Tr2*.



- For points *X* on *QL-Cu1* holds
  … *X, QA-Tf2(X), P* are collinear,
  … *CSC(X), QA-Tf2(X), CSC(P)* are collinear,
  … *CSC(X), isog(X), QL-P1* are collinear,

- Identical points:
  QA-P4 = P, *QA-P2 = CSC(P)*, *QA-P9 = QL-P1*.

- Identical lines:
  QA-P1.QA-P6 = QL-L1,
  QA-L1 = QA-L6 = CSC(P).QL-P1
  = QA-P1.QA-P2.QA-P3.QA-P9,
  QA-L2 = P.CSC(P) = QA-P2.QA-P4.QA-P6.

- Loci of *QA*-points, changing *P* on *QL-Cu1*:
  QA-P1, QA-P6, QA-P28, QA-P29 on *QL-L1*,
  QA-P3, QA-P4 on *QL-Cu1*.

Finally we study the *QG*-component of *QA*, which has not an angle bisector at *P** as sideline (see L_1, L_2 above).

- Identical points:

QG-P19 = QA-P4 = P,

QG-P1 = QG-P6 = QA-P2 = CSC(P).

- Loci of *QG*-points, changing *P* on *QL-Cu1*:

QG-P2 = QA-P29, QG-P7 = QG-P9 = QA-P1, QG-P12
on *QL-L1*,
QG-P5 = QG-P15 = QA-P3, QG-P16, QG-P18,
on *QL-Cu1*.

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