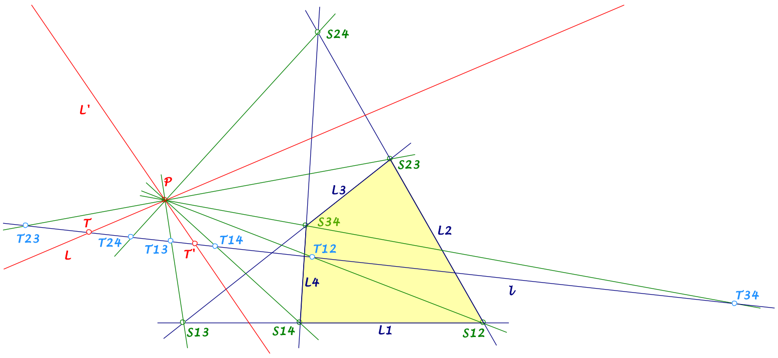
EQF-Note 2016-01-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line Transformation wrt a Point on the Line

*Background for this QL-transformation is **D**esargues **I**nvolution **T**heorem, see QA-Tf1, here used in the dual form for QL-geometry. This leads to a line transformation wrt a point on the line. The transformation maps a line L through a point P into the 2nd tangent from P to the inscribed QL-conic tangent to L.*



The transformation

L through P → L´ through P: DIT(L,P) = L´

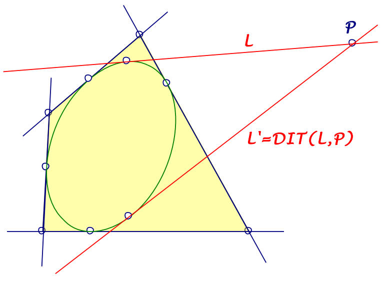
A quadrilateral with its four lines L_i has six intersections $S_{ij} = L_i \cap L_j$. For a point P (not on L_i) there are six lines $P.S_{ij}$, intersecting an arbitrary line l in six points T_{ij}. The three reciprocal pairs T_{ij}, T_{kl} define a line involution on l. Let L be a line through P and T its intersection with l and $T´$ its image wrt the line involution on l. Then the image line of L wrt P is $PT´$ = $L´$ through P with *DIT(L´,P) = L*.

Example: *DIT(QL-L9,QL-P23) = QL-P8.QL-P23*.

The point P can also be the point at infinity P_∞ of the line L:

Example: *DIT(QL-L9, P_∞) = QL-P13.P_∞*.

- ***DIT(L,P)* is the 2nd tangent from *P* at the *QL*-inscribed conic tangent to *L*.**



Lines *L* through *QL-P1*

- If *L* is a line through *QL-P1* and *P* = *QL-P1*, the line *L´* is the reflection of *L* in the 1st Steiner axis.

- If *L* is a line through *QL-P1* and *P* another intersection of *L* with *QL-Cu1*, the line *L´* is parallel *QL-L1*.

- For lines *L* = *P.QL-P1* the locus of *P* with *DIT(L,P)* = *L* is the cubic of *QFG*-message 1215.

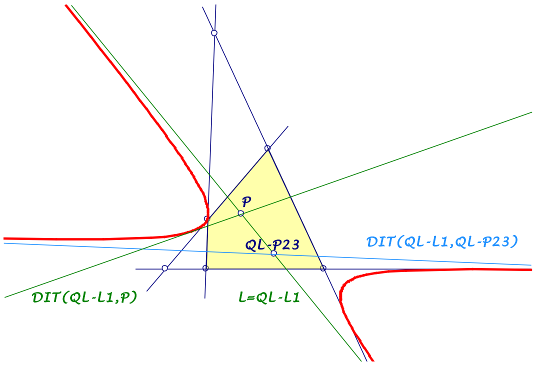
- In general: For a given point *Q* and *L* = *PQ* the locus of *P* with *DIT(L,P)* = *L* is the cubic of *QFG*-message 1175.

The property *DIT(L,P) = L*

- *DIT(L,P)* = *L*,
  … if *L* is tangent in *P* at an *QL*-inscribed conic.
  … if *P* is the intersection of *L* and *QL-Tf2(L)*.
  … if *L* is angle bisector at *P* ∈ *QL-Cu1* wrt two opposite *QL*-points.
  … if *L* = *PQ* and *P*, *Q*, *CSC(Q)* collinear on *QL-Cu1*.

Points *P* on a fixed line *L*

- *L* = *QL-L1* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope the conic *QL-Co2* with asymptote *QL-L1* and center *QL-P23*. The 2nd asymptote is *DIT(QL-L1,QL-P23)*.



- *L* = *QL-L2* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope a *QL*-inscribed conic tangent to *QL-L2* with center *QL-P20*.

- *L* = *QL-L3* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope the inscribed parabola *QL-Co1*.

- In general: Let *L* be a given line and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope a *QL*-inscribed conic tangent to *L*.

Last not least

- For two different points P_1, P_2 with $DIT(P_1P_2,P_1) \cap DIT(P_2P_1,P_2) = Q$ holds: $DIT(QP_1,Q) = P_2Q$ and $DIT(QP_2,Q) = P_1Q$.
- Let P and $P^* = CSC(P)$ be contact points of tangents at *QL-Cu1*, which intersect in Q on *QL-Cu1*:

$$DIT(PP^*,P) = PQ,$$
$$DIT(QP,Q) = QP^*,$$
$$DIT(PQ,P) = PP^* = DIT(P^*Q,P^*).$$

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