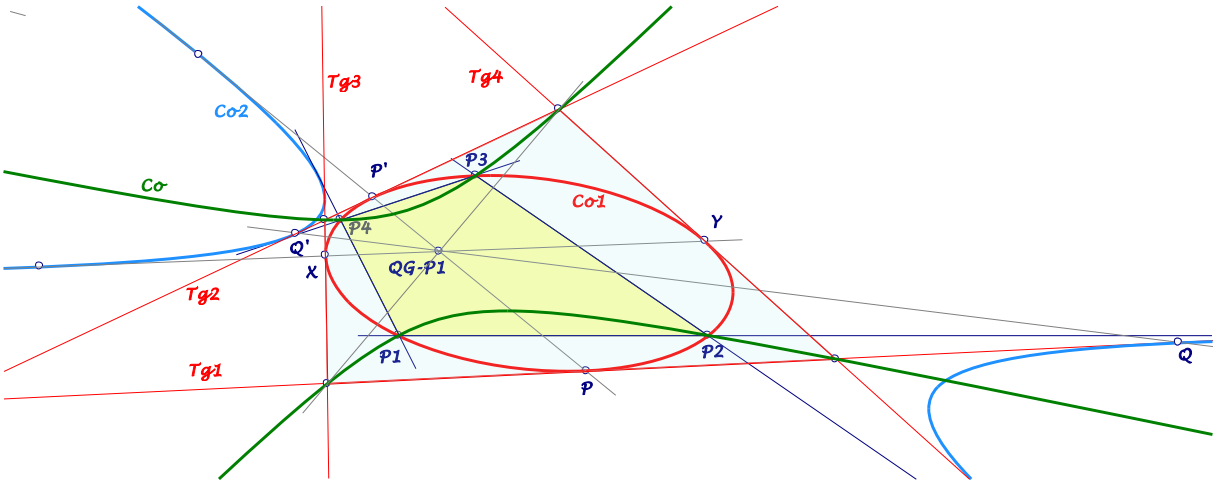
EQF-Note 2016-01-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Three QG-conics wrt a Point

For a given point P there is a QG-circumscribed conic Co_1, and the tangent in P defines an inscribed QG-conic Co_2. The four common tangents of the two conics give a quadrigon, which has a common circumscribed conic Co with the reference quadrigon. There are two QG-circumconics, whose points P give the same conic Co.



- Let $QG = P_1P_2P_3P_4$ be a quadrigon and P an arbitrary point (not on sidelines and diagonals).
- Let Co_1 be the QG-circumconic through P …
  … and Tg_1 the tangent in P at Co_1.
- Let Co_2 be the QG-inscribed conic tangent to Tg_1 …
  … with contact point Q.
- Let $P´$ be the 2nd intersection of $P.QG\text{-}P1$ and Co_1 …
  … and $Q´$ the 2nd intersection of $Q.QG\text{-}P1$ and Co_2.
- $Tg_1 = PQ$ and $Tg_2 = P´Q´$ are common tangents of Co_1 and Co_2.
- $PP´$ is the 2nd tangent from P to Co_2 …
  … with a contact point as 4th harmonic point of P, $P´$, $QG\text{-}P1$.
- Let Tg be the 2nd tangent from $QG\text{-}P1$ to Co_2 …
  … intersecting Co_1 in X and Y …
  … and contact point on Co_2 as 4th harmonic point of X, Y, $QG\text{-}P1$.
- The tangents Tg_3 and Tg_4 in X and Y at Co_1 are common tangents of Co_1 and Co_2.

- The quadrigon of the common tangents of Co_1 and Co_2 (opposite sides Tg_1, Tg_2 and Tg_3, Tg_4) has the same diagonal crosspoint *QG-P1* and the same 3rd diagonal *QG-L1* as the reference quadrigon.

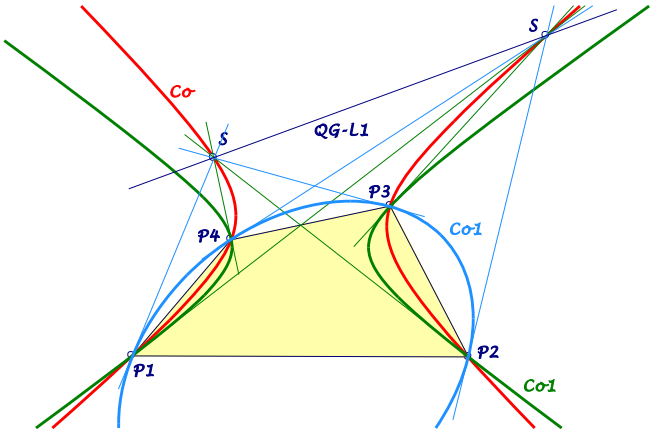
- **The quadrigon of the common tangents of Co_1 and Co_2 has not only a common inscribed conic Co_2 with the reference quadrigon but also a common circumscribed conic *Co* with the reference quadrigon:**

  _Co_ is a *QG*-circumconic through the poles of the diagonals wrt Co_1.

- **All points *P* on a fixed *QG*-circumscribed conic have the same conic *Co*.**

- **There are two *QG*-circumscribed conics, whose points *P* have the same conic *Co*:**

  Let a given *QG*-circumconic *Co* intersect *QL-L1* in *S*: A first *QG*-circumconic is tangent to $S.P_1$ and $S.P_3$, a second is tangent to $S.P_2$ and $S.P_4$.



- Example: For *Co* = *QG-Co3* the two *QG*-circumconics for the points *P* are centered in the midpoints of the diagonals.
- Example: For *P* on *QG-Co3* the second *QG*-circumconic for *P* with the same *Co* is centered in *QG-P1.QG-P2* ∩ *QG-P12.QG-P14*.

Final remark:
There are analog relationships beginning with a quadrigon and a line, which leads to a *QG*-inscribed conic with contact point …

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