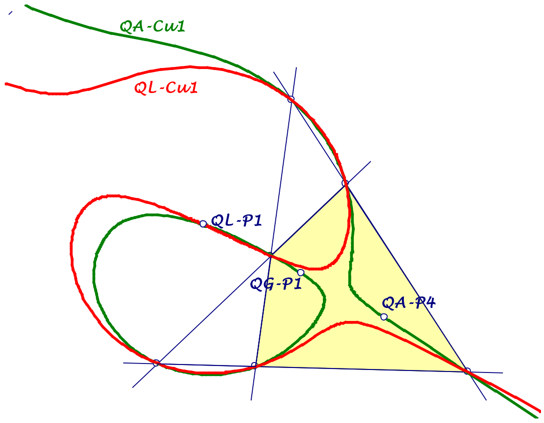
EQF-Note 2016-01-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Cu1 and QL-Cu1

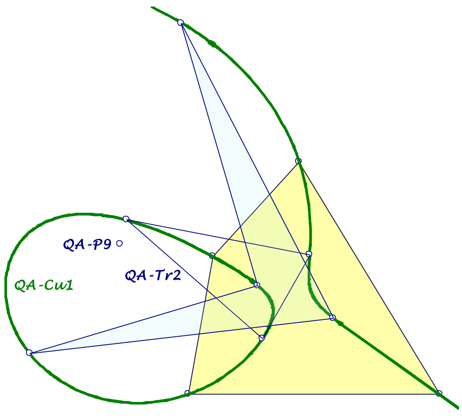
This paper looks for further quadrangles with the same cubic QA-Cu1 and for further quadrilaterals with the same cubic QL-Cu1 and the case, that the cubics coincide.



A quadrigon *QG* can be considered as quadrangle *QA* and as quadrilateral *QL*. The *QA*-cubic *QA-Cu1* and the *QL*-cubic *QL-Cu1* are *QG*-circumcubics, containing the Miquel point *QL-P1* and the intersections of opposite *QG*-sides. The cubic *QA-Cu1* is always bipartite and holds further the *QG*-diagonal crosspoint *QG-P1* and the point *QA-P4*. *QA-Cu1* is isogonal invariant wrt the Miquel triangle *QA-Tr2*. *QL-Cu1* is isogonal invariant wrt the four *QL*-triangle components and invariant wrt *QL-Tf1* = *CSC*.

Quadrangles on QA-Cu1

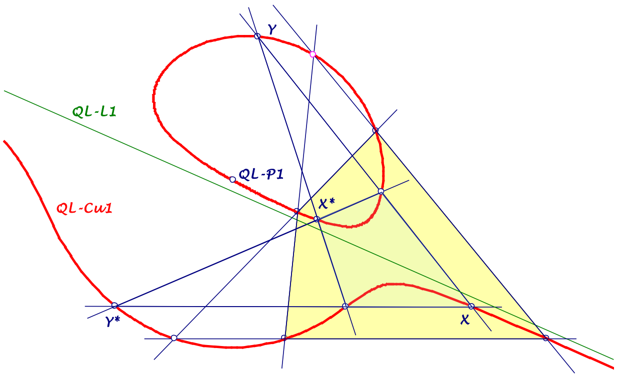
We are looking for further quadrangles on *QA-Cu1* with the same cubic *QA-Cu1*.



QA-Cu1 is invariant wrt *CSC*-analog transformations of the Miquel triangle *QA-Tr2*, which are centered in a vertex and swapping the two other vertices. A point on *QA-Cu1* and its images wrt these three transformations gives a quadrangle with the same cubic *QA-Cu1*. All these quadrangles have the same point *QA-P9*.

Quadrilaterals on QL-Cu1

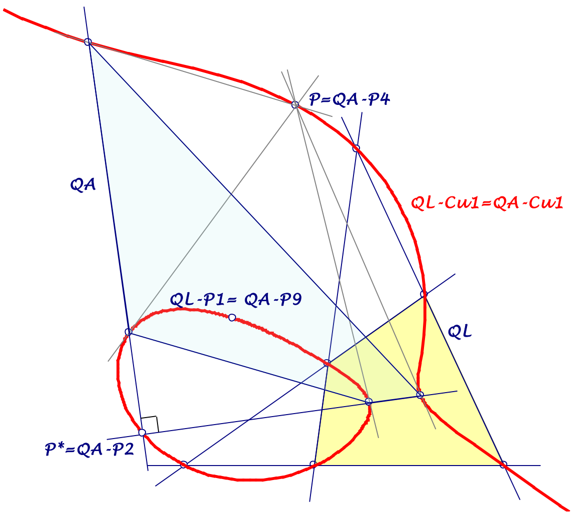
Now we are looking for further quadrilaterals on *QL-Cu1* with the same cubic *QL-Cu1*.



QL-Cu1 is *CSC*-invariant: *P* → *P**. Let a line intersect *QL-Cu1* in the points *X* and *Y*, then the quadrilateral *XY*, *XY**, *X***Y*, *X***Y** has the same cubic *QL-Cu1*. These quadrilaterals have the same Miquel point *QL-P1* and the same Newton line *QL-L1*.

QL-Cu1 as QA-Cu1

Let *QL* be a quadrilateral and *QL-Cu1* its cubic. We look for a quadrangles *QA* on *QL-Cu1*, whose cubic *QA-Cu1* is *QL-Cu1*. This is already described in *QFG*-message 1377. For a point *P* on *QL-Cu1* the contact points of tangents to *QL-Cu1* give a *QA*, whose *QA-Cu1* is *QL-Cu1* of *QL*.



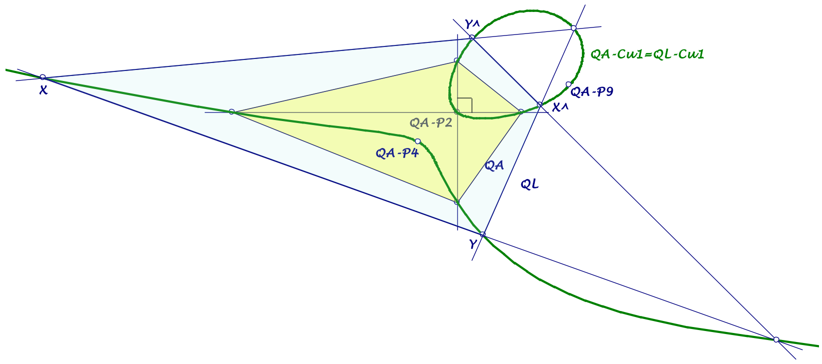
These *QA* are only real, if *QL-Cu1* is bipartite. They are special quadrangles, for the vertices lie on two orthogonal lines, which are the angle bisectors at *P** = *QA-P2* wrt opposite points of *QL*. *QA-P4* is the reference point *P* and *QA-P9* is the Miquel point *QL-P1*.

QA-Cu1 as QL-Cu1

Let *QA* be a quadrangle and *QA-Cu1* its cubic. We look for quadrilaterals *QL* on *QA-Cu1*, whose cubic *QL-Cu1* is *QA-Cu1*.

It seems, that this is only possible for quadrangles *QA*, whose vertices lie in pairs on orthogonal lines.

In this case *QA-P2* and *QA-P9* are points of *QA-Cu1*: *QA-P2* is the intersection of the orthogonal lines and *QA-P9* is the same for all *QA* on *QA-Cu1* with this cubic. So we can take an arbitrary *QA* on *QA-Cu1* with this cubic and consider a *CSC*-analog transformation ^, centered in *QA-P9* swapping *QA-P2* and *QA-P4* (see above). Taking a line, which intersects *QA-Cu1* in *X* and *Y*, the lines *XY*, *XY^*, *X^Y*, *X^Y^* give a *QL* with *QL-Cu1* = *QA-Cu1*.



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