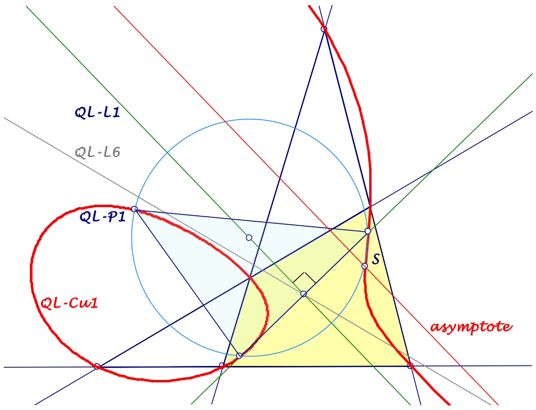
EQF-Note 2016-01-16

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Reference Triangle for QL-Cu1

If the cubic QL-Cu1 for a quadrilateral is bipartite, the cubic can be interpreted as QA-Cu1 for special quadrangles (see QFG-messages 1377, 1423). QA-Cu1 is a pivotal isogonal circular cubic wrt the Miquel triangle QA-Tr2. A corresponding reference triangle for bipartite cubics QL-Cu1 will be researched.



The Reference Triangle

For a quadrilateral *QL* with bipartite cubic *QL-Cu1* we consider a triangle *QL-Trx* with vertices …

> **…in the Miquel point *QL-P1* and the C*SC*-partners on a perpendicular line to the Newton line *QL-L1* in the intersection with *QL-L6*.**

The *CSC*-partners on the mentioned line are the intersections with *QL-Cu1*. If *QL-Cu1* is unipartite, there are no such intersections (but see the final remark).

(1) First properties of *QL-Trx* (see *QFG*-message 1379):
… The circumcircle of *QL-Trx* contains the intersection *S* of *QL-Cu1* and its asymptote diametral *QL-P1*.
… The angle bisectors at *QL-P1* are the Steiner axes.
… The in- and excenters are the intersections of the Steiner axes and *QL-Cu1* (without *QL-P1*).

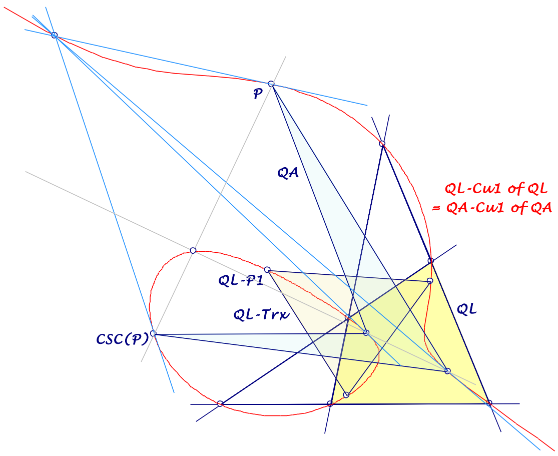
(2) *QL-Trx* as reference triangle for *QL-Cu1*:
… *QL-Cu1* is isogonal invariant wrt *QL-Trx*.

… **Isogonal conjugates on *QL-Cu1* lie on parallels to *QL-L1*.**
… ***QL-Cu1* is a pivotal isogonal cubic with reference triangle *QL-Trx* and pivot in the point at infinity of *QL-L1*.**

For the reference triangle *QL-Trx* the cubic *QL-Cu1* is not only isogonal invariant but also invariant wrt the *CSC*-analog transformations, centered in a vertex of the triangle and swapping the other two vertices. If the center is *QL-P1* this is the *CSC*-transformation of the quadrilateral (see *QL-Tf1*).

(3) *QL-Cu1* is invariant wrt the *CSC*-analog transformations for *QL-Trx*.

(4) Points on *QL-Cu1* and their *CSC*-analog images wrt *QL-Trx* have the same tangential on *QL-Cu1*.



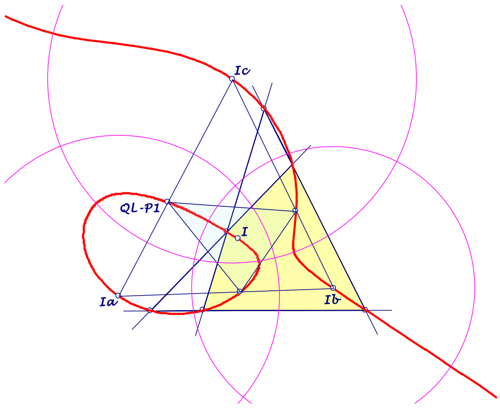
(5) A point on *QL-Cu1* and its *CSC*-analog images wrt *QL-Trx* give a quadrangle, whose Miquel triangle *QA-Tr2* is *QL-Trx* and whose cubic *QA-Cu1* is *QL-Cu1*.

These quadrangles on *QL-Cu1* have two pairs of *CSC*-partners as vertices, whose lines intersect on *QL-Cu1* and are the angle bisectors wrt opposite points of the quadrilateral.

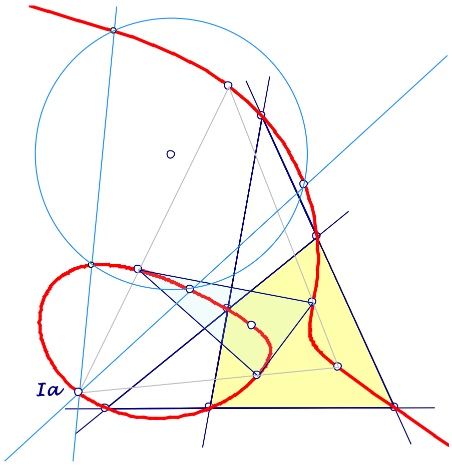
Properties of QA-Cu1 for QL-Cu1

There are a lot of properties for *QA-Cu1* in [1], which now can be translated for bipartite *QL-Cu1*. We shall use the reference triangle *QL-Trx* and the nominations of [1]:
… **Residual point** *res(P,Q)* of two points on *QL-Cu1* is the 3rd intersection of *PQ* and *QL-Cu1*.
… **Corresponding points** on *QL-Cu1* are points with the same tangential.

(6) ***QL-Cu1*** **is an anallagmatic cubic, centers of inversion are the excenters of** ***QL-Trx*****.**

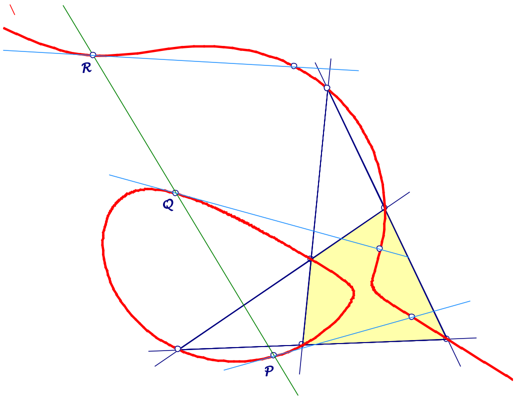


(7) Two lines through an in- or excenter of ***QL-Trx*** **intersect** ***QL-Cu1*** **concyclic.**

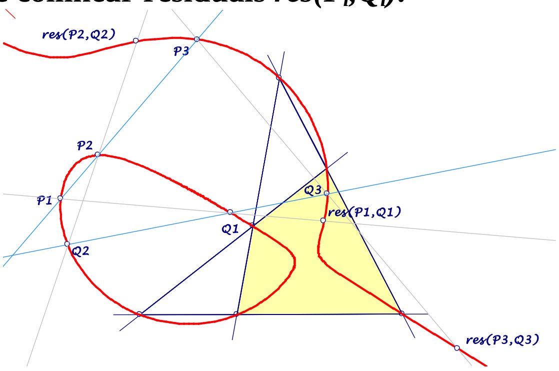


(8) The ***QL-Cu1*****-tangents in the vertices of** ***QL-Trx*** **intersect with the asymptote in** ***S*** **on** ***QL-Cu1*****.**

(9) Three collinear points on ***QL-Cu1*** **have collinear tangentials.**



(10) Two collinear Tripels P_1, P_2, P_3 and Q_1, Q_2, Q_3 on *QL-Cu1* have collinear residuals *res*(P_i,Q_i).

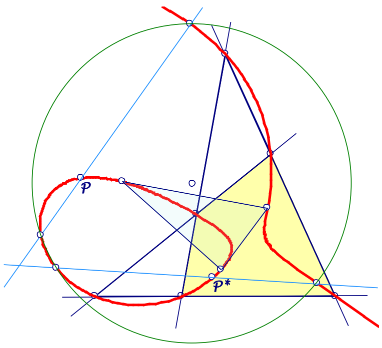


(11) For a point *P* on *QL-Cu1* the corresponding points and the residuals of *P* wrt three collinear points lie on a conic.

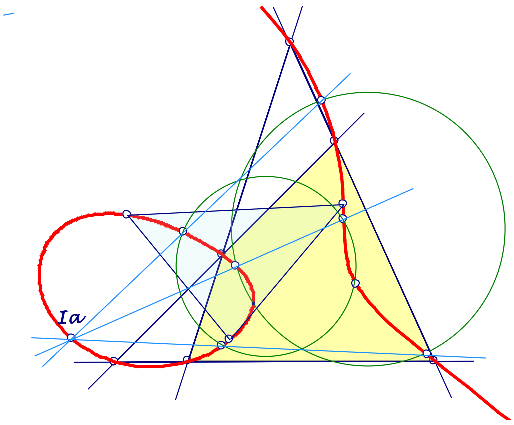
(12) The corresponding quadruples of three collinear points on *QL-Cu1* are perspective in pairs wrt points of the remaining quadruple.

(13) If two of four concyclic points on *QL-Cu1* are isogonal conjugates, the other two are collinear with the point *S*.

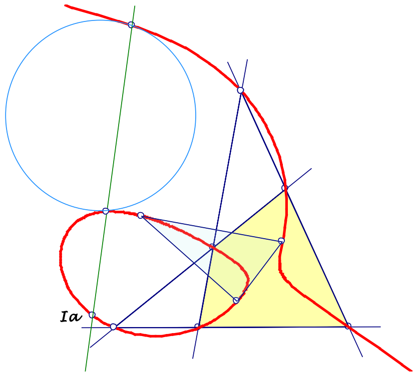
(14) Lines through isogonal points on *QL-Cu1* intersect *QL-Cu1* concyclic.



(15) The residuals of an in- or excenter of *QL-Trx* wrt four concyclic points are also concyclic.

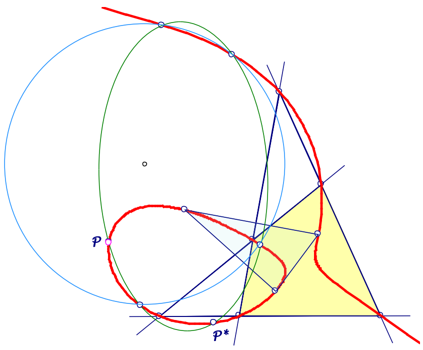


(16) The intersections of a line through an in- or excenter of *QL-Trx* and *QL-Cu1* are contact points of a circle and *QL-Cu1*.

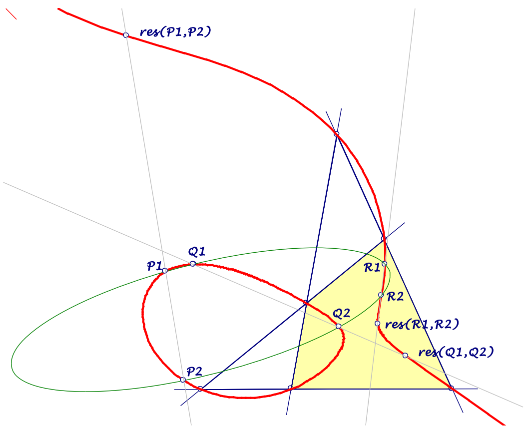


(17) A circumconic of four corresponding points on *QL-Cu1* and the common tangential contacts *QL-Cu1* in this point.

(18) A conic through two isogonal points on *QL-Cu1* has concyclic further intersections with *QL-Cu1*.

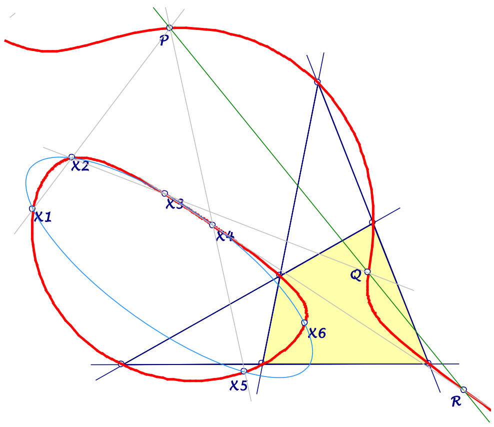


(19) The residuals for three pairs of coconic points on *QL-Cu1* are collinear.



(20) For a point X_1 and three collinear points *P, Q, R* on *CL-Cu1* the following points are coconic on *QL-Cu1*:

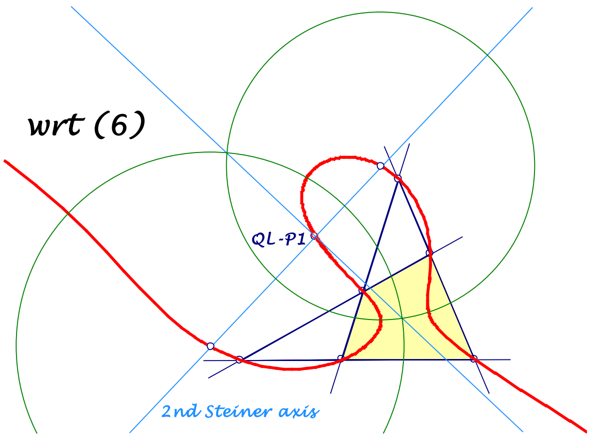
$$X_2 = res(X_1,P),\ X_3 = res(X_2,Q),\ X_4 = res(X_3,R),$$
$$X_5 = res(X_4,P),\ X_6 = res(X_5,Q),\ X_1 = res(X_6,R).$$

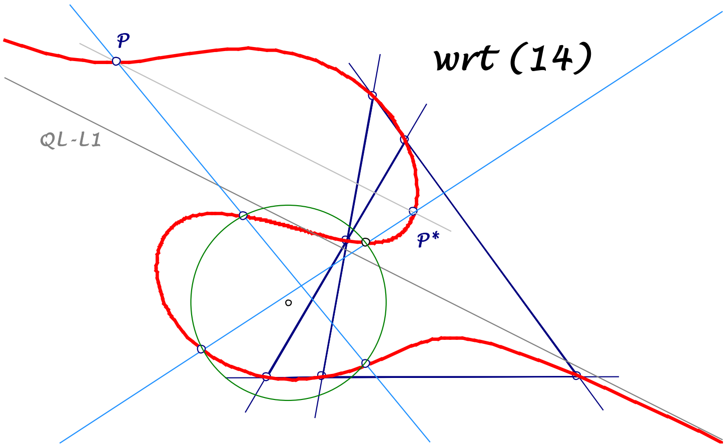


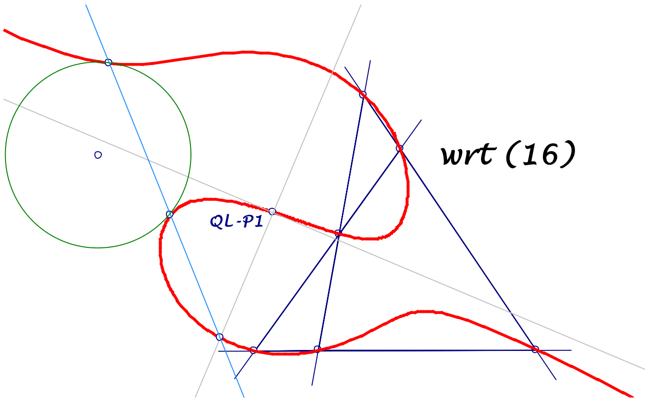
Final Remark

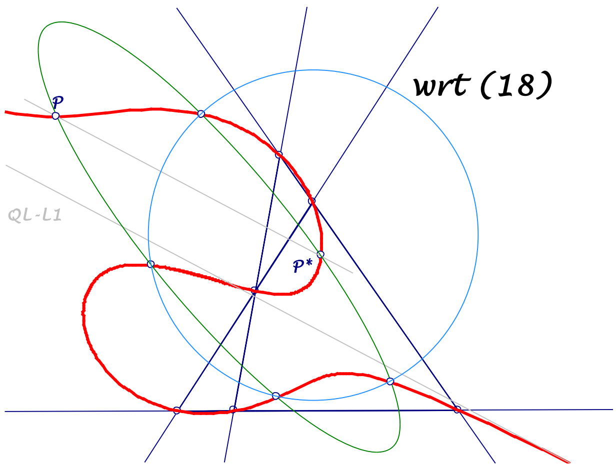
In the unipartite case of *QL-Cu1* there is no reference triangle *QL-Trx*, only the vertex *QL-P1* remains. A point on *QL-Cu1* has only one corresponding partner, which is its *CSC*-partner. Wrt the in- and excenters of *QL-Trx* in the intersections of the Steiner axes and *QL-Cu1* remain only the intersections of the 2nd Steiner axis. The isogonal conjugation wrt *QL-Trx* can be saved as 2nd intersection of a *QL-L1*-parallel and *QL-Cu1*. Wrt this background most of the properties can be hold for the unipartite case of *QL-Cu1* translating the properties in the following way:
… instead of "in- or excenters of *QL-Trx*": **intersections of the 2nd Steiner axis and *QL-Cu1***,
… instead of "isogonal conjugate wrt *QL-Trx* of *P* on *QL-Cu1*": **2nd intersection of *QL-Cu1* and a *QL-L1*-parallel through *P* .**

Examples:









Reference

[1] http://eckartschmidt.de/Zirkul..pdf

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