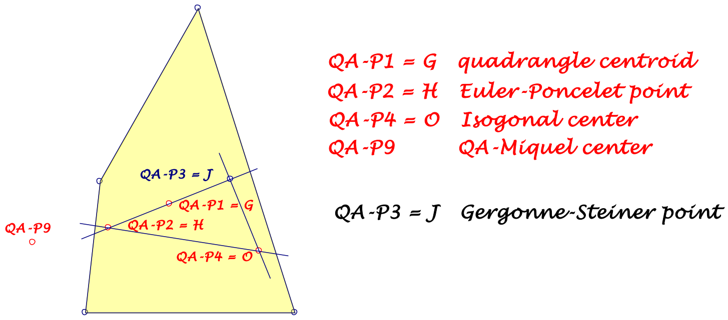
EQF-Note 2016-03-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Reconstruction of a Quadrangle

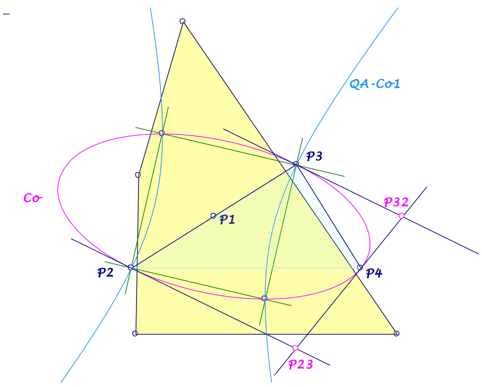
Scimemi researched in his work "Central Points of the Complete Quadrangle" (EQF-Ref.[36]) the question, how to reconstruct a quadrangle with four points. His result: G = QA-P1, H = QA-P2, O = QA-P4 and O_D = QA-P11 are such central points. Here another set is described, replacing the fourth point by the QA-Miquel Center QA-P9.



- **A quadrangle can be reconstructed from the set of central points {*QA-P1, QA-P2, QA-P4, QA-P9*}.**

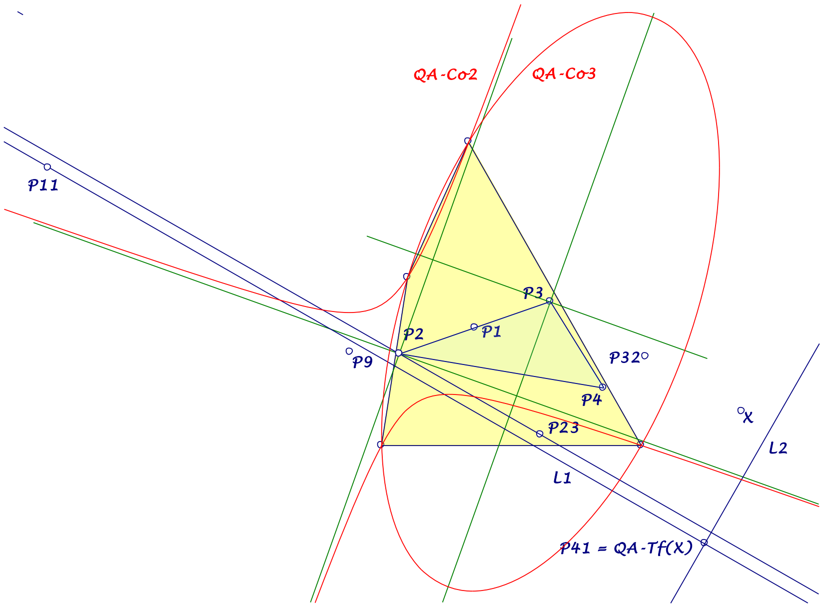
Below we shall omit the prefix *QA* for the points.

Constructions wrt *P1*, *P2*, *P4* and *P3* (reflection of *P2* in *P1*):



… The angle bisectors of <*P2.P3.P4* are
… … the axes of *QA-Co3* (centered in *P3*),
… … parallel to the asymptotes of *QA-Co2* (centered in *P2*).
… These four lines give a rectangle.
… Let *Co* be a circumscribed conic of the rectangle through *P4*.

… *Co*-tangents in *P2* and *P4* intersect in *P23*.
… *Co*-tangents in *P3* and *P4* intersect in *P32*.
… (The circumscribed conic of the rectangle, which intersects *Co* perpendicular is *QA-Co1*.)



Additional constructions with *P9* and *P23*, *P32* (see above):
… With *P9* the *QA*-Möbius conjugate *QA-Tf4* is defined, centered in *P4* swapping *P3* and *P9*.
… Let X be the reflection of *P3* in *P32*, then *QA-Tf4(X)* = *P41*.
… Let L_1 be parallel to *P2.P23* through *P41* (containing *P11*).
… Let L_2 be perpendicular to *P2.P23* through *P41*.
… *P4* and L_1 determine as pole-polar-pair *QA-Co2* (asymptotes and center see above).
… *P4* and L_2 determine as pole-polar-pair *QA-Co3* (axes and center see above).
… Intersections of *QA-Co2* and *QA-Co3* give the searched quadrangle.

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