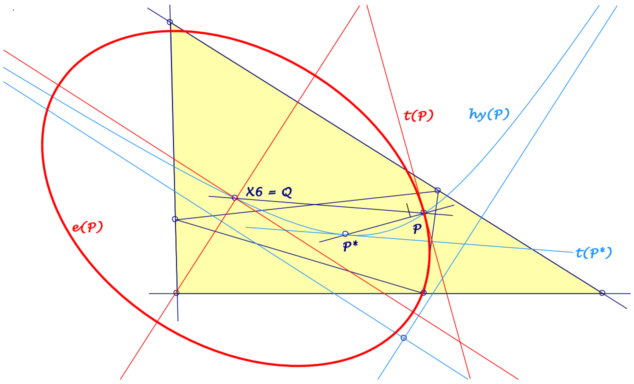
EQF-Note 2016-04-01

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Constant Sum of Distance Squares

If we consider n lines and for a point the sum of its squared distances to the lines, the locus of points with the same sum is an ellipse. A construction of this ellipse shall be described, if the point with the least sum is known. For a quadrilateral this ellipse is already researched in QFG-message 587 (EQF-Note 2014-06-04).



The construction is here only described for a triangle, but can be generalized for n lines, if the point of least squared distance sum is known. This point is the centroid of its pedal points.

- $n = 3$: Lemoine point *X(6)*,
- $n = 4$: Least Squares Point *QL-P26*,
- $n = 6$: see *QFG*-message *1589*.

Now we consider a triangle with the Lemoine point *X(6) = Q*:
… Let *P** be the centroid of the pedal triangle of *P*,
… let *t(P)* be a perpendicular line to *PP** through *P*,
… let *t(P*)* be a parallel to *PQ* through *P**,
… let *hy(P)* be an orthogonal hyperbola through *P*, *P**, *Q*, tangent to *t(P*)*.
… The parallels to the asymptotes of *hy(P)* through *Q* are the axes of the searched ellipse *e(P)*,
… which is tangent to *t(P)*.

This allows a construction of the ellipse *e(P)*. Its points have the same sum of squared distances to the triangle sidelines.

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