EQF-Note 2016-04-12

QA-Tr1 ←→ QA-Tr2 Transformation

QA-geometry can be done wrt the diagonal triangle QA-Tr1 or wrt the Miquel triangle QA-Tr2. Wrt QA-Tr1 one vertex and its anticevian triangle give the QA, wrt the Miquel triangle one vertex and its CSC-images give the QA. Here is a transformation, which swaps the vertices of QA-Tr1 and QA-Tr2. This transformation is the "Involutary Conjugate" of a related QA.
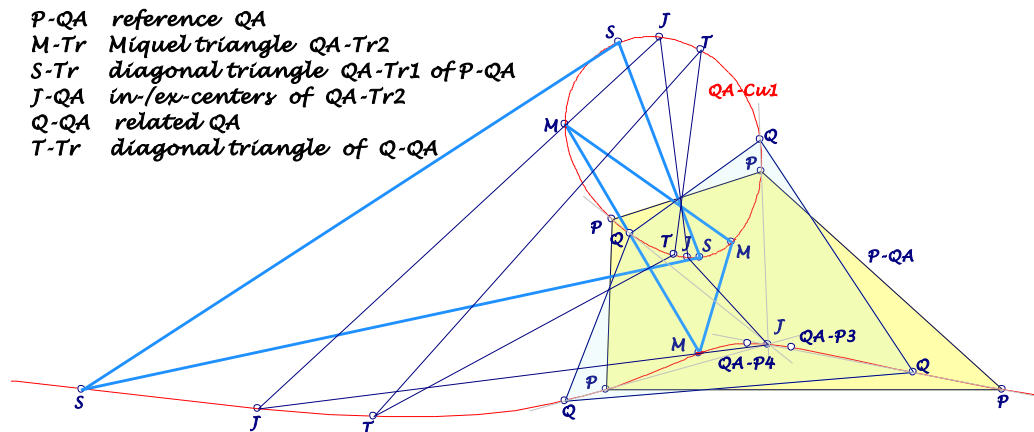

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P-QA   reference QA
M-Tr   Miquel triangle QA-Tr2
S-Tr   diagonal triangle QA-Tr1 of P-QA
J-QA   in-/ex-centers of QA-Tr2
Q-QA   related QA
T-Tr   diagonal triangle of Q-QA
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Related *QA*: Let *J-QA* be the quadrangle of the in-/ex-centers for the Miquel triangle of the reference quadrangle *P-QA*.

> **The four perspectors of *J-QA* and *P-QA* give the related quadrangle *Q-QA*.**

- **The three perspective quadrangles *P-QA*, *Q-QA*, *J-QA* have their vertices on the cubic *QA-Cu1*.**

- **The related quadrangle *Q-QA***
  … has the same Miquel triangle as *P-QA*,
  … the same cubic *QA-Cu1* as *P-QA*,
  ... its Isogonal Center is *QA-P3* of *P-QA*,
  ... its Gergonne-Steiner Point is *QA-P4* of *P-QA*.

- *QA-P3* **is perspector of the Miquel triangle and the diagonal triangle of *P-QA* (see *EQF*), *QA-P4* is perspector of the Miquel triangle and the diagonal triangle of *Q-QA*, the point at infinity of the *QA-Cu1*-asymptote is perspector of the diagonal triangles of *P-QA* and *Q-QA*.**

- The *CSC*-Tripel of *QA-P3* gives the diagonal triangle of *Q-QA*.

- The sides of the Miquel triangle and the diagonal triangles of *P-QA* and of *Q-QA* have three collinear points on a line *L*:
  … Wrt the Miquel triangle the line *L* is the trilinear polar of the midpoint of *QA-P4.QA-P3*.
  … Wrt the diagonal triangle of *P-QA* the line *L* is the trilinear polar of the reflection of *QA-P4* in *QA-P3*.
  … Wrt the diagonal triangle of *Q-QA* the line *L* is the trilinear polar of the reflection of *QA-P3* in *QA-P4*.

New *QA*-transformation *QA-Tfx:* Let *QA-Tfx* be the Involutary Conjugate of the related quadrangle *Q-QA*, an isoconjugation wrt the diagonal triangle of *Q-QA* with fixed points in the vertices of *Q-QA*.

- **The Involutary Conjugate of the related quadrangle *Q-QA* swaps the vertices of the diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2*.**

- The cubic *QA-Cu1* is invariant wrt *QA-Tfx*.

- The cubic *QA-Cu1* is a pivotal isocubic wrt
  … reference triangle: diagonal triangle of *Q-QA*,
  … isoconjugation: *QA-Tfx*,
  … pivot: *QA-P3*.

- *QA-Tfx(QA-P4)* is the point at infinity of *QA-Cu1*.

- *QA-Tfx(QA-P3)* is the 3rd intersection of *QA-Cu1* and the line, connecting *QA-P41* and the intersection of *QA-Cu1* and its asymptote.

- *QA-Tfx(Pi)* are the 3rd intersections of *QA-Cu1* and *Pi.QA-P3*.

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