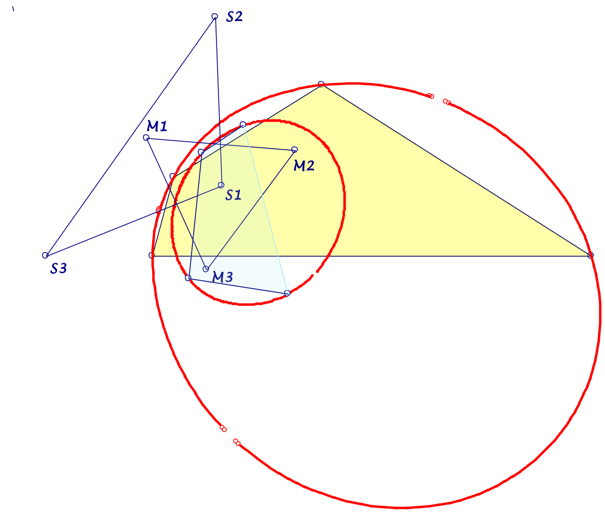
EQF-Note 2016-04-17

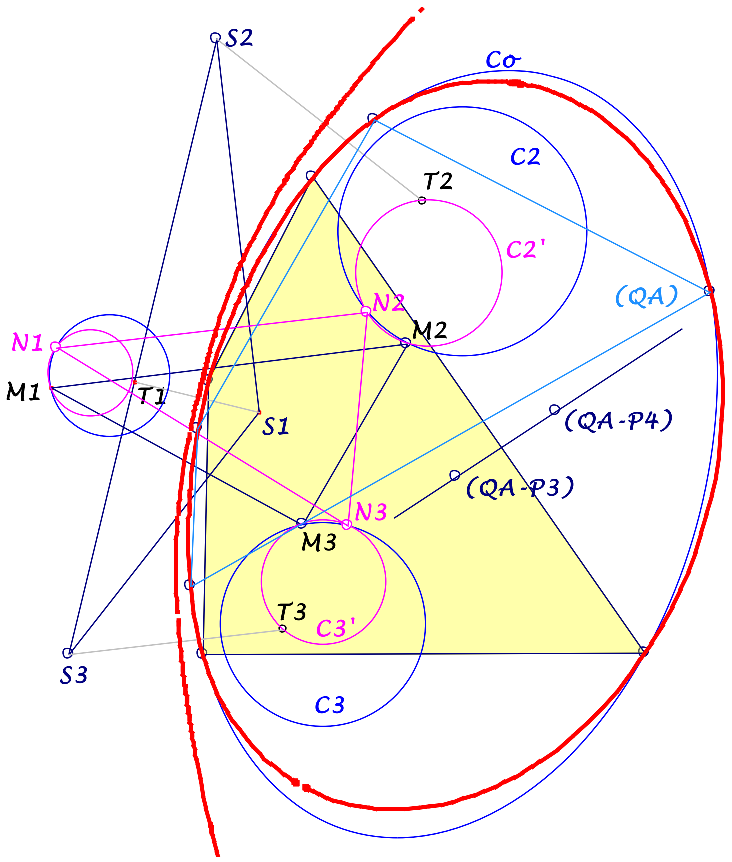
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Sigma-Quartic

QA-P4 is the homothetic center of the 1st circumcenter quadrangle and the 1st isogonal conjugate quadrangle. The corresponding ratio is Stärk´s Sigma of a quadrangle in:
Roland Stärk: Eine merkwürdige Zahl des Vierecks.
PM: Praxis der Mathematik in der Schule, 46 (2004) 1, S. 26-31.
This parameter also appears as "lambda" in the paper of Benedetto Scimemi (EQF-ref.[36], 3.2.3 to 3.2.6).
Stärk's Sigma of a quadrangle has a lot of interesting properties, see also EQF-message 1471. In this message a quartic is mentioned for the vertices of quadrangles with the same diagonal triangle and the same Sigma. Here for this quartic a construction is described.



For a reference quadrangle *QA*
… let $S_1S_2S_3$ be the diagonal triangle *QA-Tr1*,
… $M_1M_2M_3$ the Miquel triangle *QA-Tr2* and
… $T_1T_2T_3$ the triangle of the pedal points of S_i wrt S_jS_k.
… Let C_i be the circles of Apollonius for M_i wrt *SiTi*.
… Let *Co* be a circumconic of *QA* and *P* points on *Co*,
… *P* and its *QA-Tr1*-anticevians give a *P-QA* on *Co* (not in the figure).
… The Miquel points of these *P-QA* give circles $C_i´$
… through M_i and T_i
… with 2nd intersections N_i with C_i.



Wrt the searched quadrangle brackets are used: The triangle $N_1N_2N_3$ will be the Miquel triangle *(QA-Tr2)* of the searched quadrangle *(QA)*. For this quadrangle we know its diagonal triangle *QA-Tr1* and its Miquel triangle *(QA-Tr2).* The perspector of these triangles is *(QA-P3)*. The isogonal conjugate of *(QA-P3)* wrt *(QA-Tr2)* is *(QA-P4)*. The involutary conjugate *(QA-Tf2)* of *(QA-P3)* is the point at infinity of *(QA-P3).(QA-P4)*. The fixed points of the isoconjugation *(QA-Tf2)* are the vertices of *(QA)*. The construction of a quadrangle, using the diagonal triangle and the Miquel triangle and knowing a pair of points wrt the involutary conjugate, is described in *QFG*-message 1679.

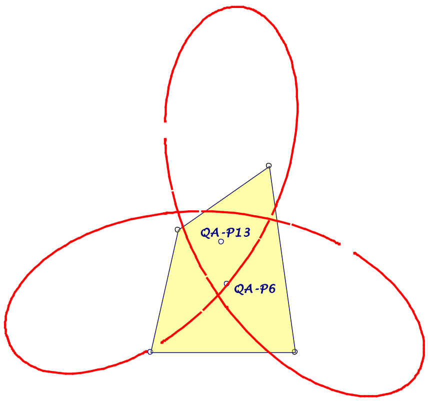
Changing the *QA*-circumconic *Co*, we will get the searched quartic in the vertices of *(QA).*

Some properties:

- The quartic is *QA*-circumscribed.
- The quartic contains with a point also its *QA-Tr1*-anticevians.
- Let a *QA*-line P_iP_j intersect the quartic also in Q_1,Q_2, then the line involution wrt these pairs of points has double points in a *QA-Tr1*-vertex and in the intersection with the opposite *QA-Tr1*-sideline.
- A line S_iS_j cuts each part of the quartic harmonically.
- A line S_iT_i cuts each part of the quartic harmonically.

It is interesting, to study the loci of *EQF*-points for quadrangles *(QA)* on the quartic. Some examples beside the evident constellations wrt the common diagonal triangle:

Wrt ***QA-P4*** : Circle (through *QA-P4*) round *QA-P12*.
… ***QA-P6*** : See final figure.
… ***QA-P8*** : Conic (through *QA-P8*), centered on *QA-L5* …
… ***QA-P23***: Conic (through *QA-P23*), centered in *QA-P11* …
… ***QA-P28***: Circle (through *QA-P28*) round *QA-P13*.
… ***QA-P35***: Circle (through *QA-P35*), centered in a point, dividing *QA-P11.QA-P12* with ratio 2:3 and radius 1/5 of the *QA-Tr1*-circumcircle. This circle is the locus of all 1st Penta Points for an arbitrary point and its *QA-Tr1*-anticevians.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de