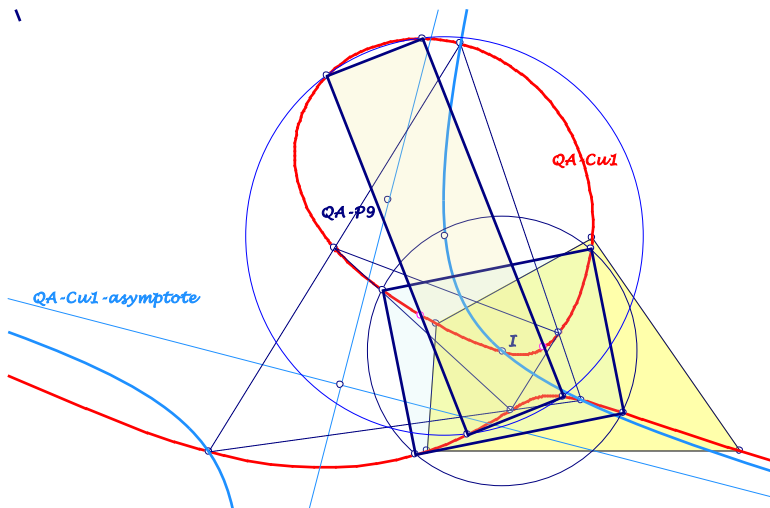


Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Rectangles on QA-Cu1 and QL-Cu1

In QFG-message 1773 a special rectangle on the cubic QA-Cu1 of a quadrangle is described, but there is a set of inscribed rectangles, also for the cubic QL-Cu1 of a quadrilateral. Here the loci for the centers of these rectangles are described, which are orthogonal hyperbolas. The results are CABRI-observations, a general construction for the circumcircles of the rectangles cannot be given.



Rectangles on QA-Cu1

For a triangle there is a circle round the incenter I , whose inversion swaps a vertex and the reflection of the corresponding excenter in the incenter. If we consider this circle for the Miquel triangle of a quadrangle, its intersections with the cubic $QA-Cu1$ give a rectangle (see *QFG-message 1773*). The inversion of $QA-Cu1$ wrt this circle is the reflection in the incenter of the Miquel triangle.

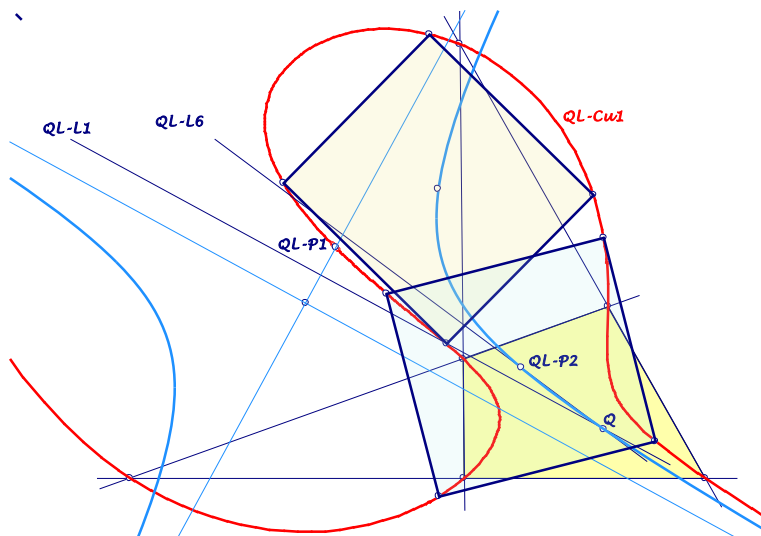
But there are more rectangles inscribed $QA-Cu1$ (see figure above):

- **The centers of rectangles on $QA-Cu1$ lie on an orthogonal hyperbola**
 - ... through the in-/ex-centers of the Miquel triangle $QA-Tr2$,
 - ... with center in the pedal point of $QA-P9$ on the asymptote of $QA-Cu1$,
 - ... which is also asymptote of the hyperbola.

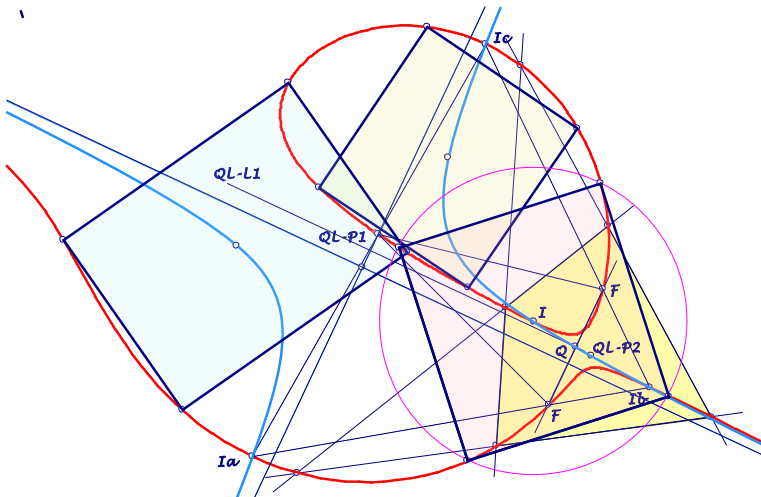
Rectangles on $QL-Cu1$

If $QL-Cu1$ of a quadrilateral is bipartite, it can be considered as $QA-Cu1$ of a special quadrangle (see *QFG*-message 1377) and the observations above hold for $QL-Cu1$ in a corresponding way. But we will give a description in QL -geometry.

- The centers of rectangles on $QL-Cu1$ lie on an orthogonal hyperbola,
 - ... through $QL-P2$ and the intersection Q of $QL-L1$ and $QL-L6$,
 - ... centered in the reflection of $QL-P1$ in the Newton line $QL-L1$,
 - ... with an axis parallel $QL-L1$.



- In the bipartite case of $QL-Cu1$ the orthogonal hyperbola of the rectangle centers
 - ... bears the in-/ex-centers of the triangle with vertices $QL-P1$ and the foci F of a QL -inscribed conic centered in $Q = QL-L1 \cap QL-L6$.



If we consider for the described triangle the circle, mentioned at the beginning, the intersections with $QL-CuI$ give the corresponding rectangle. Analog: The inversion wrt this circle gives for $QL-CuI$ the reflection in I .

On the orthogonal hyperbolas there are only special regions for the rectangle centers. A general construction of the circumcircles of the rectangles cannot be given.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de