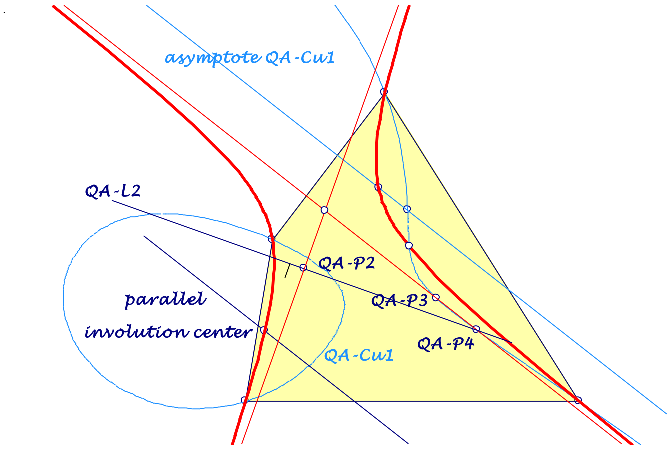
EQF-Note 2016-08-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Circumscribed Hyperbolas wrt QA-Tf1

The QA-Line Involution QA-Tf1 marks on a line an involution center (see EQF). For a line pencil this transformation gives a curve, for example QA-Cu6 for the pencil of QA-P1. If we take the line pencil of a point at infinity P_∞, that means a set of parallels, the locus of the involution centers is a QA-circumscribed hyperbola.



First example

Let P_∞ be the point at infinity of the line L = *QA-P3.QA-P4*, which is a parallel to the asymptote of *QA-Cu1*.

- **The involution centers of parallels to the line *QA-P3.QA-P4* give a**
  … *QA*-circumscribed hyperbola,
  … centered in the *QA-Tf2*-image of the point at infinity of a perpendicular to *QA-P2.QA-P23*,
  … one asymptote *QA-P3.QA-P4*,
  … the other asymptote a perpendicular to *QA-L2* through *QA-P2*,
  … intersecting *QA-Cu1* in the isogonal conjugate of *QA-P41* wrt the Miquel triangle *QA-Tr2*.

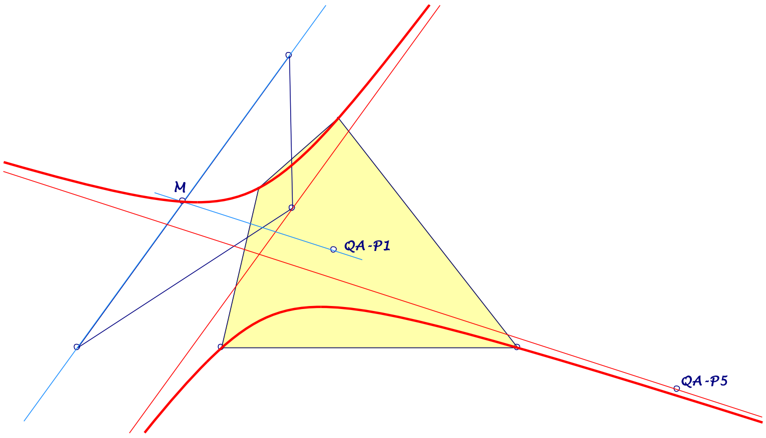
The equation of this hyperbola in *DT*-notation is:

$$q^2 r^2 \left(q^2 (c^2 p^2 - a^2 r^2)^2 - r^2 (b^2 p^2 - a^2 q^2)^2\right) x^2$$
$$+ r^2 p^2 \left(r^2 (b^2 p^2 - a^2 q^2)^2 - p^2 (c^2 q^2 - b^2 r^2)^2\right) y^2$$
$$+ p^2 q^2 \left(p^2 (c^2 q^2 - b^2 r^2)^2 - q^2 (c^2 p^2 - a^2 r^2)^2\right) z^2 = 0$$

- **In general: The point at infinity of the parallels is the point at infinity of one asymptote of the involution center hyperbola. Parallels to the other asymptote give the same involution center hyperbola.**

Second example

- **Parallels to a side of the diagonal triangle *QA-Tr1* give an involution center hyperbola**
  … circumscribed the quadrangle through the midpoint *M* of the *QA-Tr1*-side,
  … one asymptote parallel to the *QA-Tr1*-side through the opposite *QA-Tr1*-vertex,
  … the other asymptote parallel to *M.QA-P1* through *QA-P5*.



Final remark

- **The infinity points of the axes of the two quadrangle parabolas give *QA-Co1* as involution center hyperbola, for these axes are parallel to the asymptotes of *QA-Co1*.**

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