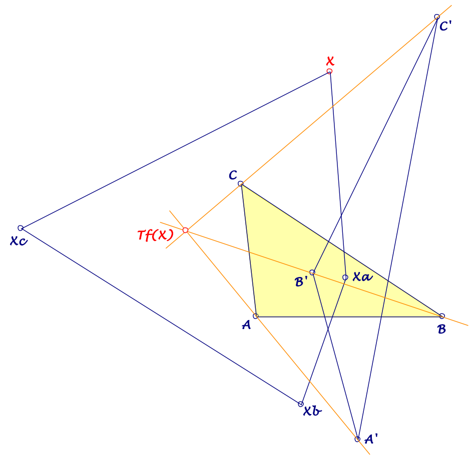
EQF-Note 2016-09-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Cu1 Invariant Transformation

Hidden in Bernard Gibert´s paper
"Perspective Triangles
Inscribed in the Neuberg Cubic"
there is a triangle transformation, not involutary, but mapping the Neuberg Cubic to itself. This transformation can be developed with QA-geometry and maps – wrt the Miquel triangle – the cubic QA-Cu1 to itself.



Consider for a reference triangle *ABC* and a point *X*
… the images X_a, X_b, X_c of *X* wrt
…the Möbius transformations, centered in a vertex and swapping the other two vertices (see *QA-Tf4*).
…The diagonal triangle of the quadrangle $XX_aX_bX_c$ is perspective to the reference triangle in *Tf(X).*
The first coordinate in barycentric coordinates is:

$$(c^2 x^2 y + b^2 x^2 z + 2 a^2 x y z + a^2 y^2 z + a^2 y z^2) / \\ (b^2 c^2 x^4 + 2 b^2 c^2 x^3 y - a^2 c^2 x^2 y^2 + b^2 c^2 x^2 y^2 + c^4 x^2 y^2 - \\ 2 a^2 c^2 x y^3 - a^2 c^2 y^4 + 2 b^2 c^2 x^3 z + 4 b^2 c^2 x^2 y z - \\ 2 a^2 c^2 y^3 z - a^2 b^2 x^2 z^2 + b^4 x^2 z^2 + b^2 c^2 x^2 z^2 + a^4 y^2 z^2 - \\ a^2 b^2 y^2 z^2 - a^2 c^2 y^2 z^2 - 2 a^2 b^2 x z^3 - 2 a^2 b^2 y z^3 - a^2 b^2 z^4)$$

This is the transformation $M \rightarrow M\sim*$ in 2.2 of Bernard Gibert's paper, only considered for the Neuberg cubic. Properties should be read there. Not mentioned a generalization:

- **All pivotal isogonal circular cubics are invariant wrt *Tf*.**

The cubic *QA-Cu1* is a pivotal isogonal circular cubic wrt the Miquel triangle *QA-Tr2*:

- ***QA-Cu1* is invariant wrt *Tf* for the Miquel triangle.**

There are always four points on *QA-Cu1* with the same *Tf*-image:

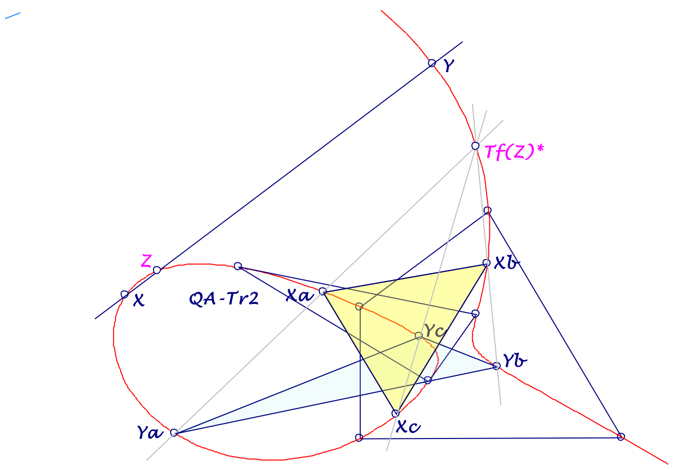
- **The *Tf*-images of the quadrangle vertices are *QA-P3*.**

- **The *Tf*-images of the vertices of the diagonal triangle and *QA-P4* are the isogonal conjugate of *QA-P41*.**

- **The *Tf*-images for the in- and excenters of *QA-Tr2* are the intersection of *QA-Cu1* and its asymptote.**

- **The *Tf*-images of *QA-Cu1*-points with the same tangential are the isogonal conjugate of the tangential.**

The last property gives a simple construction for the tangential *Tf(X)** of a *QA-Cu1*-point *X*.
The following property is an analogon to chapter 3 in Bernard Gibert's paper:



- **For two *QA-Cu1*-points *X, Y***
  … the diagonal triangles of $XX_aX_bX_c$ and $YY_aY_bY_c$ are perspective
  … at the tangential of the third intersection *Z*: *Tf(Z).**

Example: Let X = *QA-P3* and Y = *QA-P4*: The third intersection Z is the point at infinity of *QA-Cu1* with tangential in the intersection of *QA-Cu1* and its asymptote, which is the perspector of $X_aX_bX_c$ and $Y_aY_bY_c$.

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