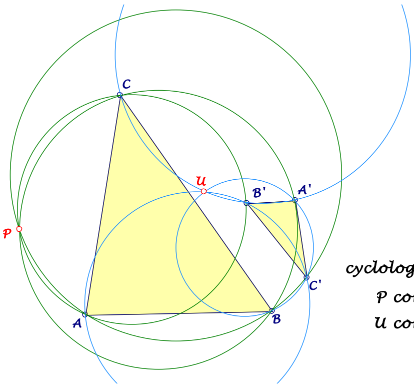
EQF-Note 2016-10-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Generalized Cyclologic centers

This is an idea of Tsihong Lau
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/3275
here reduced to a starting constellation of two cyclologic triangles and their cyclologic centers. Background are cyclologic QA-triple triangles (see QA-Tr-4).

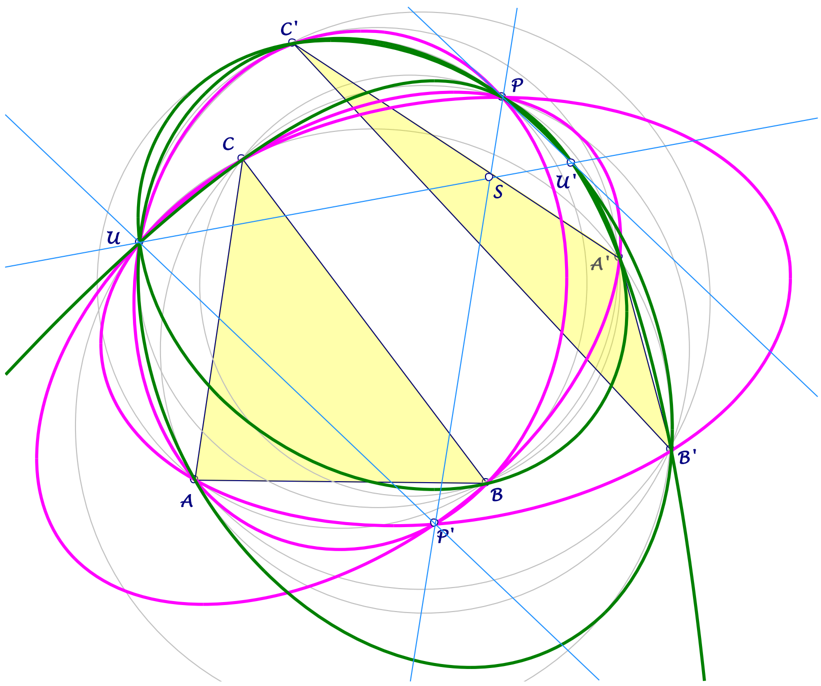


cyclologic centers of ABC and A'B'C'
P concyclic ABC', AB'C, A'BC
U concyclic A'B'C, A'BC', AB'C'

Let *ABC* and *A´B´C´* be two cyclologic triangles with centers *P* and *U* as in the figure above.
Tsihong Lau consideres **6 conics**
… circumscribed *ABC´*, *AB´C*, *A´BC*, *A´B´C*, *A´BC´*, *AB´C´*,
… through *P* and *U*
… with a further intersection in *P´* for the first 3 conics
… and a further intersection in *U´* for the last 3 conics.
The points *P´* and *U´* shall here be named the first generation of generalized cyclologic centers.
For triangles, which are symmetric wrt a point and therefore cyclologic, the first generation of generalized cyclologic centers doesn´t exist. The six conics are identical.
For the cyclologic centers *P*, *U* and the generalized cyclologic centers *P´*, *U´* of the first generation holds

- ***PU´* and *P´U* are parallel.**
- **The Möbius transformation centered in *PP´*∩*UU´* swapping *P* and *U* swaps also *P´* and *U´*.**

Normally this cannot be generalized for further generations. But let us consider cyclologic *QA*-triple triangles as described in *QA-Tr-4*. For *QA-P1*-symmetric pairs *QG-P1/QG-P15*, *QG-P4/QG-P8*, *QG-P5/QG-P10*, *QG-P7/QG-P9* the *QA*-triple triangles as well as their cyclologic centers are symmetric wrt *QA-P1*, so there are no generalized cyclologic centers.

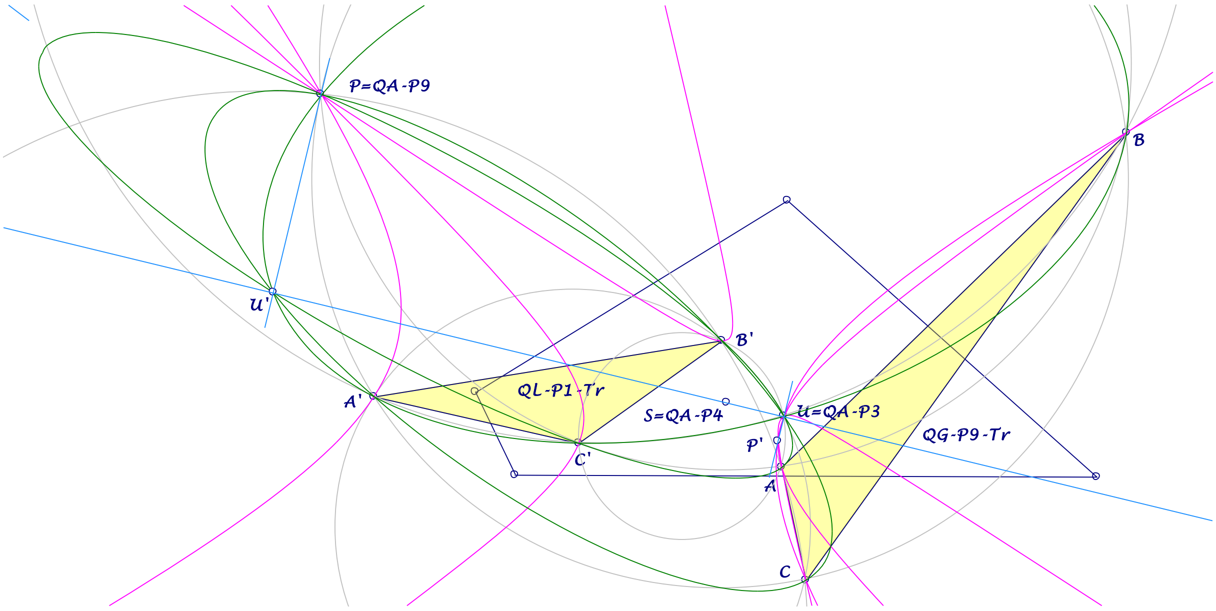


Example 1: Let
… *ABC* be the Miquel triangle *QA-Tr2*,
… *A´B´C´* a component triangle $P_iP_jP_k$
… with *P* = perspector of $P_iP_jP_k$ and *QA-Tr2*, *U* = *QA-P4*,
… … *P´*= *QA-P3*, *U´* = P_l.

Example 2: Let wrt ***QG-P1/QL-P1***
… *ABC* be the diagonal triangle *QA-Tr1*,
… *A´B´C´* the Miquel triangle *QA-Tr2*
… with *P* = *QA-P41*, *U* = *QA-P3*, *P´* = *QA-P4*, *U´* = *QA-P41** (*QA-Tr2*-isogonal conjugate *).

Example 3: Let wrt ***QG-P5/QL-P1***
… *ABC* be the *QG-P5*-triangle,
… *A´B´C´* the Miquel triangle *QA-Tr2*
… with *P* = *QA-P9*, *U* = *QA-P3*, *S* = *QA-P4*,
… … *P´* = intersection of *QA-P4.QA-P9* and a perpendicular to *QA-P3.QA-P4* in *QA-P3*,
… … *U´* = intersection of *QA-P4.QA-P3* and a perpendicular through *QA-P9*.

- **For the cyclologic *QA*-triple triangles of *QG-P5* and *QL-P1* the transformation *QA-Tf4* swaps the cyclologic centers *P* = *QA-P9* and *U* = *QA-P3* as well as *P´* and *U´*, *P´´* and *U´´*, …**



There are other examples, that the Möbius transformation swaps also the generalized cyclologic centers of further generations:

- **For the cyclologic *QA*-triple triangles wrt**
  QG-P1/QG-P18, QG-P1/QG-P19, QG-P1/QL-P1, QG-P1/QL-P17, QG-P5/QL-P1, QG-P5/QL-P4, QG-P9/QL-P1, QG-P9/QL-P4, QG-P18/QG-P19, QL-P1/QL-P17
  the Möbius transformation, centered in *PP´*∩*UU´* and swapping the cyclologic centers *P*, *U*, swaps not only the generalized cyclologic centers *P´*, *U´* of the first generation, but also those of further generations.

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