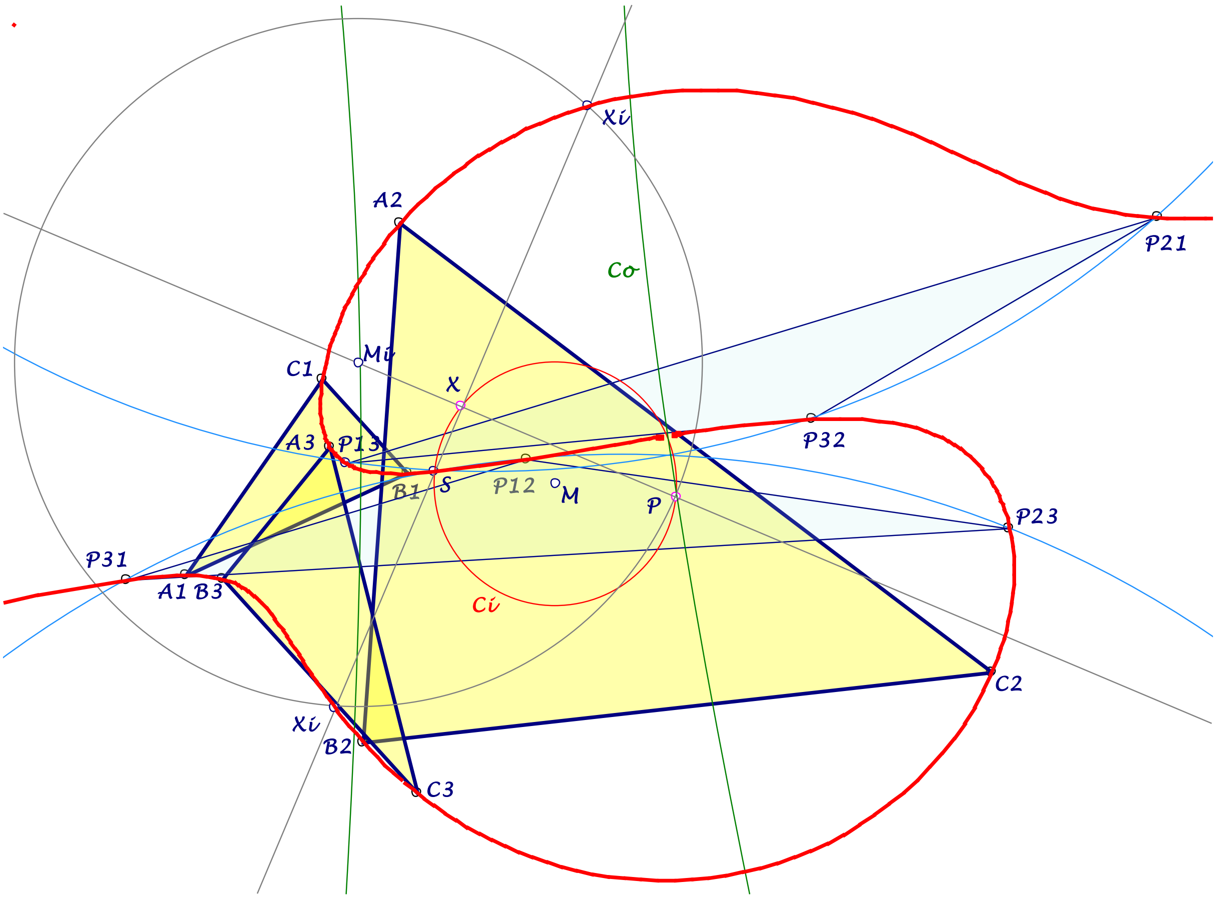
EQF-Note 2016-11-17

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubics of Generalized Cyclologic Centers II

Generalized cyclologic centers are defined by Tsihong Lau [1]. Starting with two cyclologic triangles and their cyclologic centers the generalized cyclologic centers give a cubic (see #1988, #1990 and #1994). Here a construction of this cubic is given for the general case.



Two triangles $A_1B_1C_1$ and $A_2B_2C_2$ are cyclologic, if the circles $(A_1B_1C_2)$, $(B_1C_1A_2)$, $(C_1A_1B_2)$ have a common point, the cyclologic center P_{12}. The circles $(A_2B_2C_1)$, $(B_2C_2A_1)$, $(C_2A_2B_1)$ then have also a common point, the cyclologic center P_{21}.
There are generations of generalized cyclologic centers, the first generation is:
$P_{12}´$ = 3rd common point of the conics $(A_1B_1C_2P_{12}P_{21})$, $(B_1C_1A_2\,P_{12}P_{21})$, $(C_1A_1B_2\,P_{12}P_{21})$,
$P_{21}´$ = 3rd common point of the conics $(A_2B_2C_1P_{12}P_{21})$, $(B_2C_2A_1\,P_{12}P_{21})$, $(C_2A_2B_1\,P_{12}P_{21})$.

Cabri observations show, that the locus of these generalized cyclologic centers is a cubic, which now shall be constructed.

For two cyclologic triangles there is a third triangle $A_3B_3C_3$ with the vertices

$$A_3 = B_1C_2 \cap B_2C_1,\ B_3 = C_1A_2 \cap C_2A_1,\ C_3 = A_1B_2 \cap A_2B_1.$$

These three triangles are pairwise cyclologic, so we get over all six cyclologic centers P_{12}, P_{21}, P_{13}, P_{31}, P_{23}, P_{32}.

- **The triangles $P_{12}P_{23}P_{31}$ and $P_{32}P_{13}P_{21}$ are perspective and similar.**

Let
… S be the perspector, which is one intersection of the circumcircles of the two triangles $P_{12}P_{23}P_{31}$ and $P_{32}P_{13}P_{21}$,
… M the midpoint of the circumcenters of the two triangles
… and *Ci* the circle round M through S,
… P the reflection of S in M,
… X a point on *Ci*.

For cyclologic triangles on the cubic *QA-Cu1* is
… point S = intersection of *QA-Cu1* and its asymptote,
… point P = *QA-P9*,
… circle *Ci* = the circumcircle of the Miquel triangle *QA-Tr2*.

For cyclologic triangles on the cubic *QL-Cu1* is
… point S = the intersection of *QL-Cu1* and its asymptote,
… point P = *QL-P1*,
… circle *Ci* = the *CSC*-image of a *QL-L1*-perpendicular line through the intersection with *QL-L6*.

The searched cubic bears
… the vertices of the triangles $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$,
… the cyclologic centers P_{12}, P_{21}, P_{13}, P_{31}, P_{23}, P_{32}
… and the perspector S.

- **The asymptote of the cubic is a parallel to $P_{12}P_{21}'$ or $P_{21}P_{12}'$ through S.**

Let X be a point on *Ci* and X_1 and X_2 the further intersections of XS and the cubic. Cabri observations show:

- **A line XS intersects the cubic symmetrically to X.**
- **Centers of circles through X_1, X_2, P lie on a conic *Co* through P.**

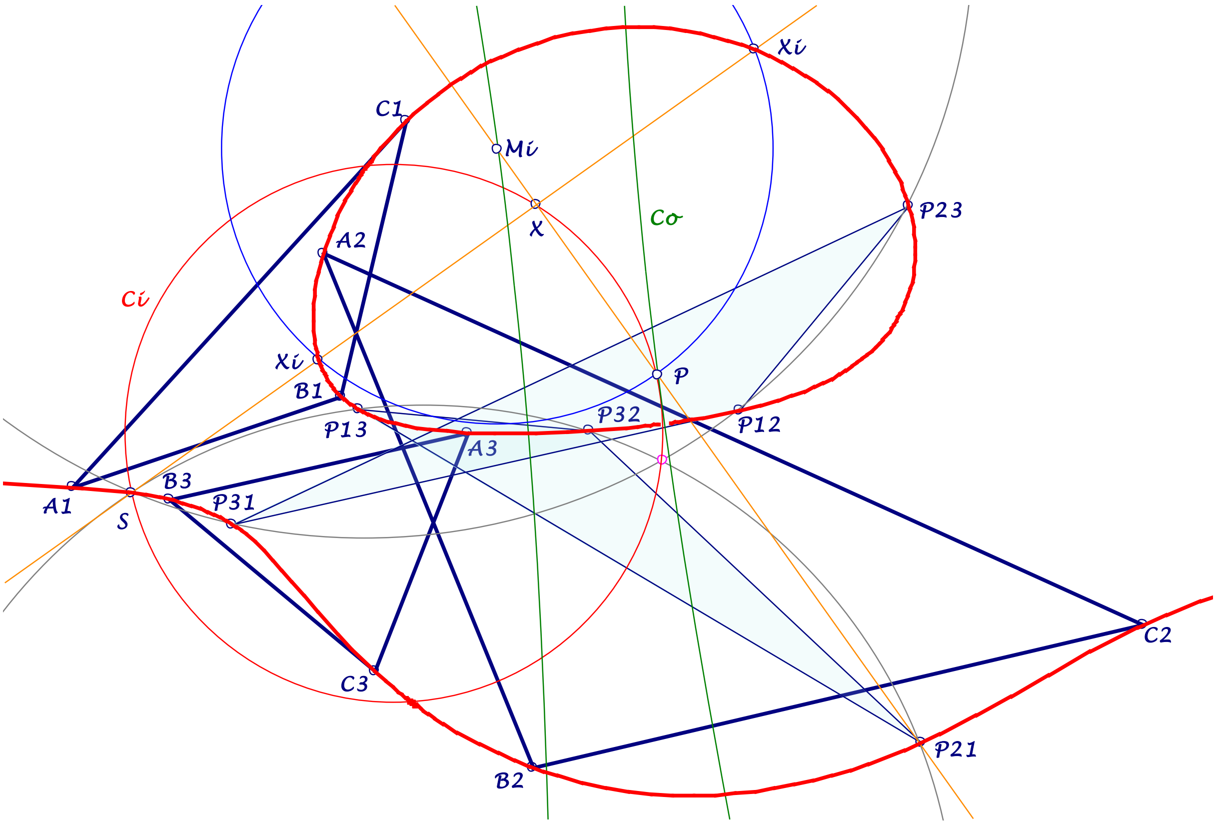
This conic *Co* can be constructed as follows:
… Let X_i be an already known point of the cubic (see above),
… let X be the 2nd intersection of X_iS and *Ci*,
… let M_i be the intersection of XP and the perpendicular bisector of X_iP,
… then 4 points M_i and the point P define the conic *Co*.

Now the cubic can be constructed:
… Let X be a variable point on Ci,
… M_i the 2nd intersection of XP and Co,
… then the circle round M_i through P intersects XS in points X_i of the cubic.

The cubic can be tested with the following property:

- **Pairs of the triangles $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$ are generalized cyclologic wrt any two points of the constructed cubic.**

Final remark: The cubic can be unipartite or bipartite. In the bipartite case the three intersections – unequal S – of the cubic and the circle Ci give a reference triangle, so that the cubic is an isogonal pivotal isocubic with pivot in the infinity point of the asymptote.



Ref [1]
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/3275

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