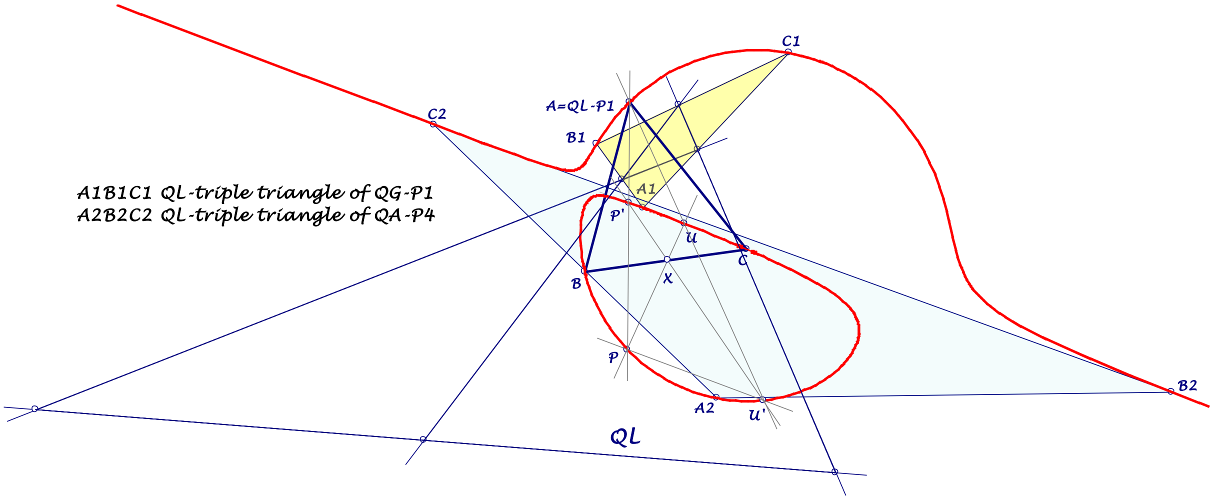
**EQF-Note 2016-11-30**

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

## QL-Triple Triangles for QG-P1 and QA-P4

*The transformation QL-Tf1 = CSC swaps QG-P1 and QA-P4 for a quadrigon. So we have for a quadrilateral cyclologic QL-triple triangles for these two points (QFG #2009). Here the corresponding cubic for the generalized cyclologic centers (QFG #2022) shall be researched.*




Let $A_1B_1C_1$ be the *QL*-triple triangle of *QG-P1*, which is the diagonal triangle *QL-Tr1* of the quadrilateral.
Let $A_2B_2C_2$ be the *CSC*-image of $A_1B_1C_1$, which is the *QL*-triple triangle of *QA-P4*.

- **The *QL*-triple triangles of *QG-P1* and *QA-P4* are cyclologic.**

Let *P* and *U* be the cyclologic centers:
*P* common point of the circles $(A_1B_1C_2)$, $(B_1C_1A_2)$, $(C_1A_1B_2)$.
*U* common point of the circles $(A_2B_2C_1)$, $(B_2C_2A_1)$, $(C_2A_2B_1)$.
Let *P´* and *U´* be the generalized cyclologic centers of the first generation:
*P´* common point of the conics $(A_1B_1C_2PU)$, $(B_1C_1A_2PU)$, $(C_1A_1B_2PU)$.
*U´* common point of the conics $(A_2B_2C_1PU)$, $(B_2C_2A_1PU)$, $(C_2A_2B_1PU)$.

- ***P, P´* and *U, U´* are collinear with *QL-P1*.**
- ***PU´* and *P´U* are parallel.**
- ***P, U* and *P´, U´* are *CSC*-partners.**
- ***P* lies on the circle $(A_2B_2C_2)$, *U* on the circle $(A_1B_1C_1)$.**

Further we consider the triangles $A_3B_3C_3$ and $A_4B_4C_4$ with …
… $A_3 = B_1C_1 \cap B_2C_2$, $B_3 = C_1A_1 \cap C_2A_2$, $C_3 = A_1B_1 \cap A_2B_2$,
… $A_4 = B_1C_2 \cap B_2C_1$, $B_4 = C_1A_2 \cap C_2A_1$, $C_4 = A_1B_2 \cap A_2B_1$.

- **$A_3B_3C_3$ is the cevian triangle of *QL-P26* wrt $A_2B_2C_2$.**
- **$A_4B_4C_4$ is the *CSC*-image of $A_3B_3C_3$.**
- **$A_3B_3C_3$ and $A_4B_4C_4$ are cyclologic triangles.**

- **The cubic for the generalized cyclologic centers bears**
  **… the vertices of the four triangles,**
  **… the cyclologic centers *P, U, P´, U´***
  **… and the Miquel point *QL-P1*.**

- **The cubic for the generalized cyclologic centers**
  **… is *CSC*-invariant**
  **… with an asymptote parallel *PU´* or *P´U*.**

The construction of the cubic is already described in *QFG*-message 2022, here another construction:
… Let *X* be the intersection of *PU* and *P´U´*,
… let *Y* be the corresponding point for the cyclologic triangles $A_3B_3C_3$ and $A_4B_4C_4$
… and *B*, *C* the intersections of *XY* and its *CSC*-circle,
… additional *A* = *QL-P1* gives a reference triangle *ABC*.

- **The cubic for the generalized cyclologic centers of the *QL*-triple triangles for *QG-P1* and *QA-P4* is**
  **… a pivotal isogonal circular cubic**
  **… with reference triangle *ABC***
  **… and pivot in the point at infinity of *PU´* or *P´U*.**

In this way the cubic can be considered as *QA-Cu1* for a quadrangle, taking a point of the cubic and its three Möbius transformations wrt *ABC*. Final result:

- **Points *D* of the cubic and the *ABC*-isogonal conjugate of *CSC(D)* are collinear with *QL-P1*.**


Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de