EQF-Note 2016-12-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Conic-Perspector QA-Co-1

In EQF we find a conic-perspector QA-Co-1, defined for circumconics Co of the QA-diagonal triangle. These conics can be considered as QA-Tf2-image of a line L, so we get a transformation CPT, which maps a line to a point. This transformation is also described by Tran Quang Hung in QFG-message 2078. Here special cubics for a line pencil wrt this transformation are researched.
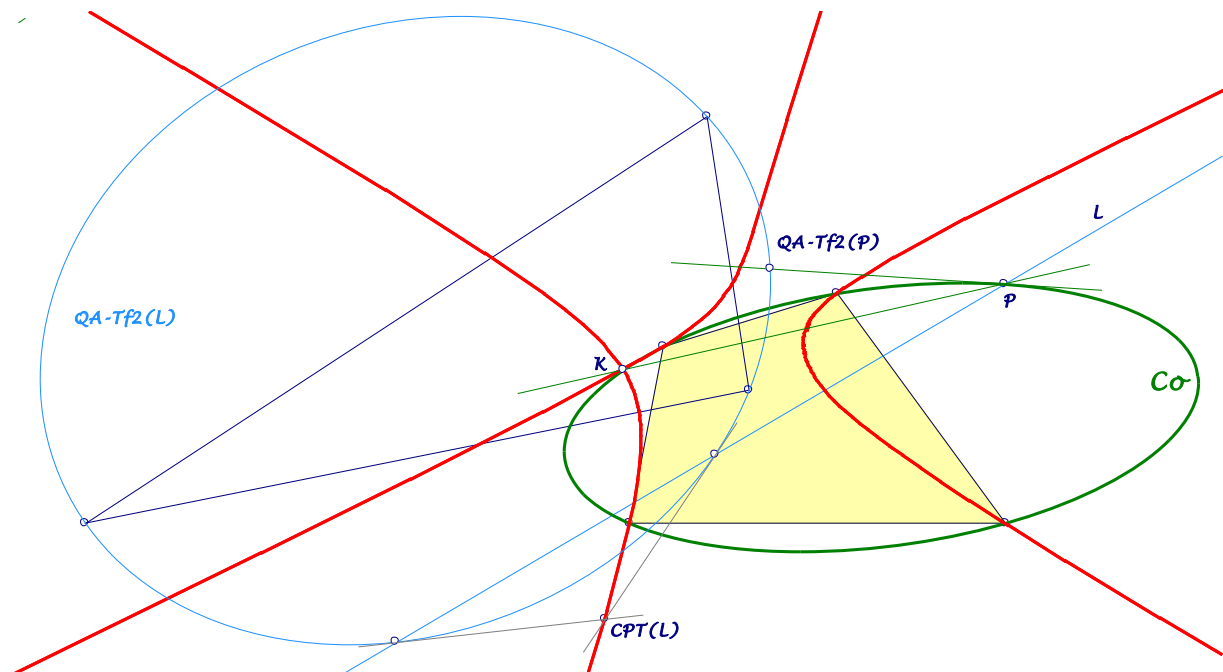
Tran Quang Hung describes his transformation as follows:
Let $P_1P_2P_3P_4$ be a quadrangle,
… L any line,
… $Pi´$ the tripole of L wrt $PjPkPl$,
… then the lines $PiPi´$ are concurrent in *CPT(L)*.
The nomination *CPT* shall be a shortcut of "Conic-Perspector of the *QA-Tf2*-Transformed line".

- ***CPT(L) is the pole of L wrt the conic QA-Tf2(L).***

Examples (see also *EQF*):
(1) *Co = QA-Ci1*,
... *L* = polar of *QA-P38* wrt *QA-Ci1*,
…*CPT(L) = QA-P38*.

(2) *Co = QA-Co1*,
… *L* = line at infinity,
… *CPT(L) = QA-P1*.
(3) *Co = QA-Co4*,
… *L = QA-L4*,
… *CPT(L)* = pole of *QA-L4* wrt *QA-Co4*.
(4) *Co = QA-Co5*,
… *L = QA-L3*,
… *CPT(L)* = pole of *QA-L3* wrt *QA-Co5*.

Now we consider the line pencil of a point *P*.

- **The *CPT*-images of lines *L* through a point *P* give a *QA*-circumcubic**
  … with knot *K*
  … … on the *QA*-circumconic *Co* through *P*
  … … as intersection – unequal *P* – of *Co* and the *Co*-polar of *QA-Tf2(P)*.

- **The 3rd intersection of *PK* and the cubic is the *CPT* of the tangent *tg* in *P* at *Co*.**
- ***CPT(PK), QA-Tf2(P)* and *K* are collinear.**

Let *ABC* be the *QA*-diagonal triangle, then *CPT(PA), CPT(PB), CPT(PC)* give a special triangle on the cubic.

- **For the *QA*-diagonal triangle *ABC* the *CPT*-images of the lines *PA, PB, PC* give the *ABC*-cevian triangle of *QA-Tf2(P)*.**
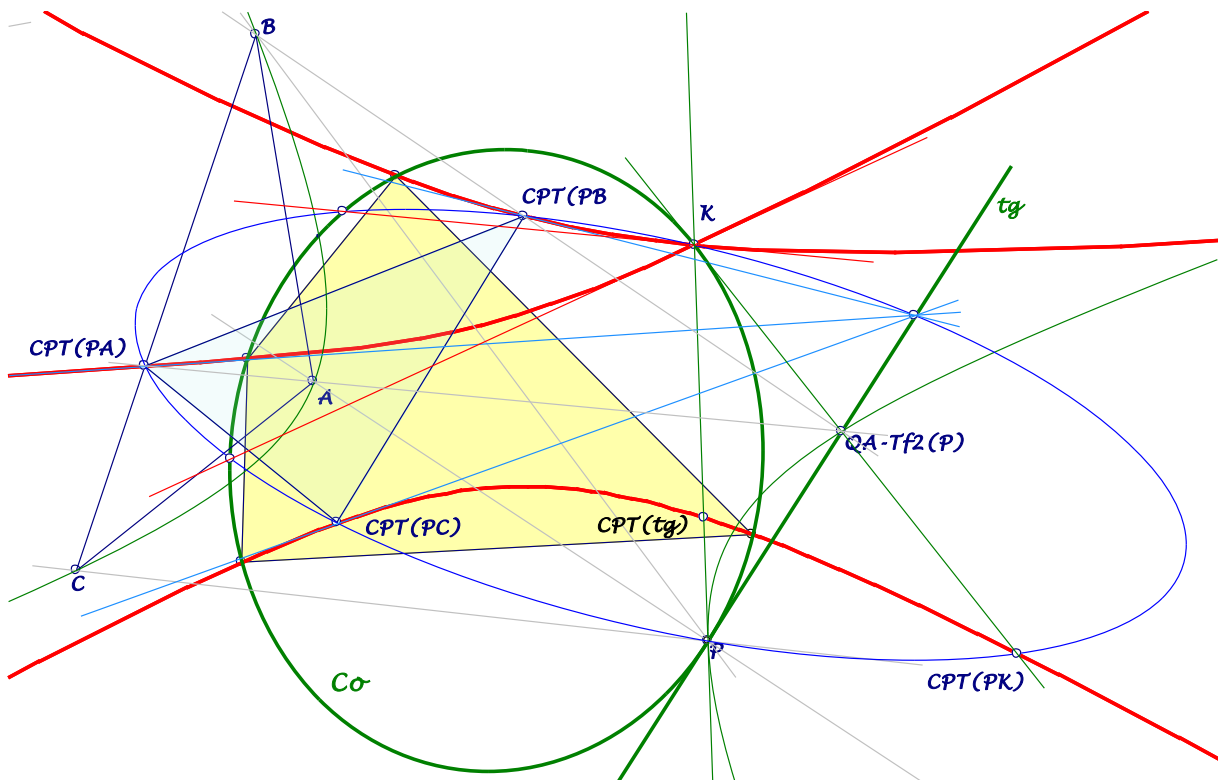
- **The tangents in *CPT(PA)*, *CPT(PB)*, *CPT(PC)* at the cubic have a common point
  … on the tangent in *P* at *Co*
  … in the 2nd intersection with the circumconic of *CPT(PA)*, *CPT(PB)*, *CPT(PC)*, *P* and *K*.**

- **The tangents in *K* at the cubic are *KX* and *KY* with *X*, *Y* 3rd and 4th intersection of the conics
  … *Co* through the *QA*-vertices and *P*
  … and the circumconic of *CPT(PA)*, *CPT(PB)*, *CPT(PC)*, *P* and *K*.**

Example for *P* = *QA-P6* with *QA*-circumconic *Co* through *P*:
… *CPT* of a perpendicular in *QA-P6* wrt *QA-L2* is *QA-P38*,
… tangent in *P* at *Co* is *QA-P6.QA-P30*,
… knot *K* is the 2nd intersection of *Co* and the polar of *QA-P30*,
… *CPT(PK)* is collinear with *K* and *QA-P30*.
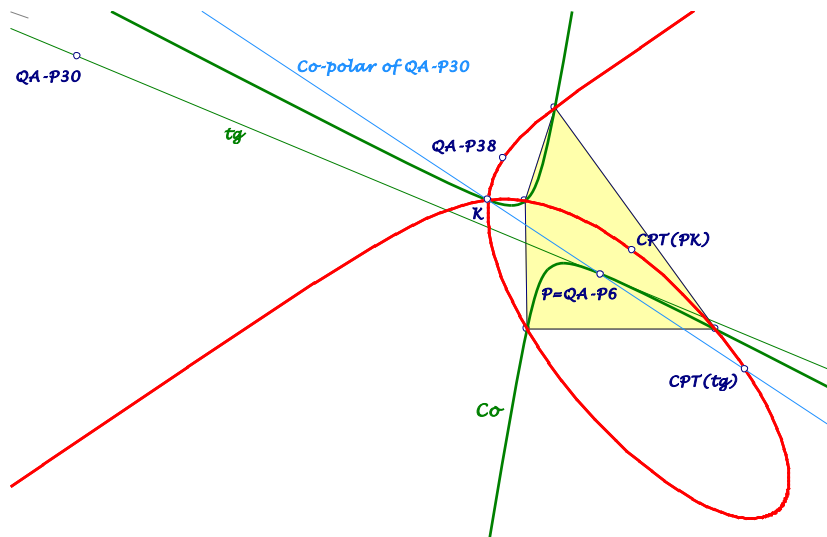
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