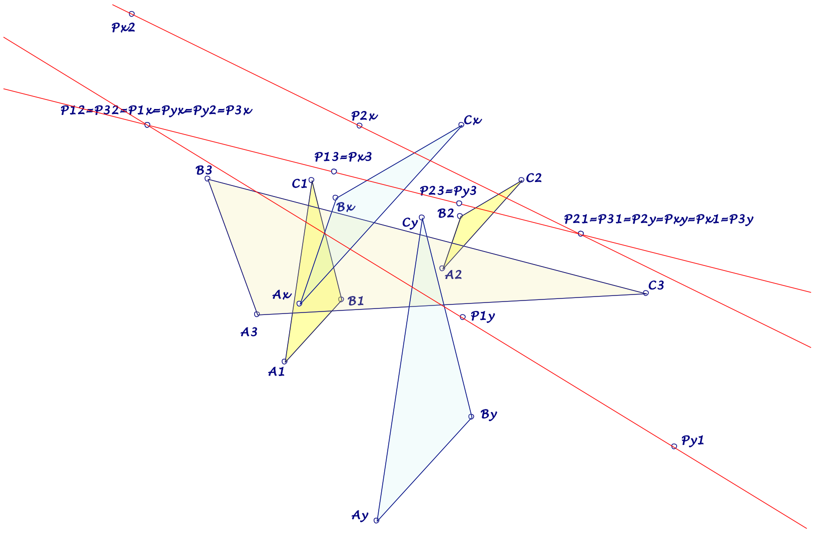
EQF-Note 2017-03-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Orthologic QA-Triple Triangles

Two orthologic QA-triple triangles have a lot of further orthologic QA-triple triangles. Here two aspects are researched. Of interest are the generating QG-points and the orthologic centers of these triangles.



(1) Third Orthologic Triangle

Let $A_1B_1C_1$ and $A_2B_2C_2$ be two orthologic triangles with centers P_{12} and P_{21} (see *QFG*-message *2354*):

- **The 3rd orthologic triangle $A_3B_3C_3$ of two orthologic triangles $A_1B_1C_1$ and $A_2B_2C_2$ is P_{12}-perpective wrt $A_1B_1C_1$ and P_{21}-perspective wrt $A_2B_2C_2$.**

- **The three triangles are pairwise orthologic and for the centers holds**
  $P_{12} = P_{32}$, $P_{21} = P_{31}$, P_{13}, P_{23} are collinear.

- **For two orthologic *QA*-triple triangles wrt the points Q_1 and Q_2 the 3rd orthologic triangle is the *QA*-triple triangle of $Q_3 = Q_1.P_{12} \cap Q_2.P_{21}$.**

Example: Let Q_1 = *QG-P*$_1$, Q_2 = *QG-P9*
… Q_3 = *QL-L*$_1$ ∩ *QL-P*$_1$.*QG-P*$_1$,
… $P_{12} = P_{32}$ = *QA-P3*, $P_{21} = P_{31}$ = *QA-P32*,
… P_{13} = 2nd intersection of *QA-P3.QA-P32* and *QA-Co4*,
… P_{23} = 2nd intersection of *QA-P3.QA-P32* and the orthogonal hyperbola for the *QG-P9*-triple triangle through *QG-P32*.

There are two further triangles, orthologic to $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$:
… $A_xB_xC_x$ = 3rd orthologic triangle of $A_1B_1C_1$ and $A_3B_3C_3$,
… $A_yB_yC_y$ = 3rd orthologic triangle of $A_2B_2C_2$ and $A_3B_3C_3$.
The 3rd orthologic triangle of $A_xB_xC_x$ and $A_yB_yC_y$ is $A_3B_3C_3$.

- **The five triangles $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$, $A_xB_xC_x$, $A_yB_yC_y$ are pairwise orthologic.**

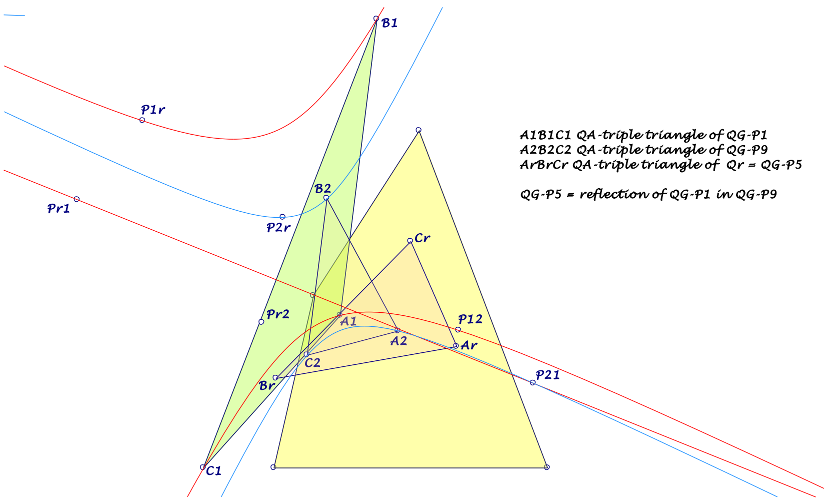
Wrt the orthologic centers see the figure above. Further properties can be found in *QFG*-message *2356*, *2357*, *2361* with correction in message *2364*.

(2) Orthogonal *QA*-Triple Triangles for collinear points

In *QFG*-message *2378* there is another possibility, to get further orthologic *QA*-triple triangles:

Let Q_1 and Q_2 be *QG*- or *QL*-points with orthologic *QA*-triple triangles $A_1B_1C_1$ and $A_2B_2C_2$ (see *QA-Tr-3*) and Q_r a point, dividing Q_1Q_2 with ratio *r* (independent of the quadrangle) and generating a *QA*-triple triangle $A_rB_rC_r$.

- **The *QA*-triple triangles of Q_1, *Q2*, Q_r are pairwise orthologic.**



- **The loci for the orthologic centers are:**
… for P_{1r}: orthogonal circumhyperbola of $A_1B_1C_1$ through P_{12},
… for P_{r1}: line through P_{21} and *X(4)* of $A_1B_1C_1$,
… for P_{2r}: orthogonal circumhyperbola of $A_2B_2C_2$ through P_{21},
… for P_{r2}: line through P_{12} and *X(4)* of $A_2B_2C_2$.

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