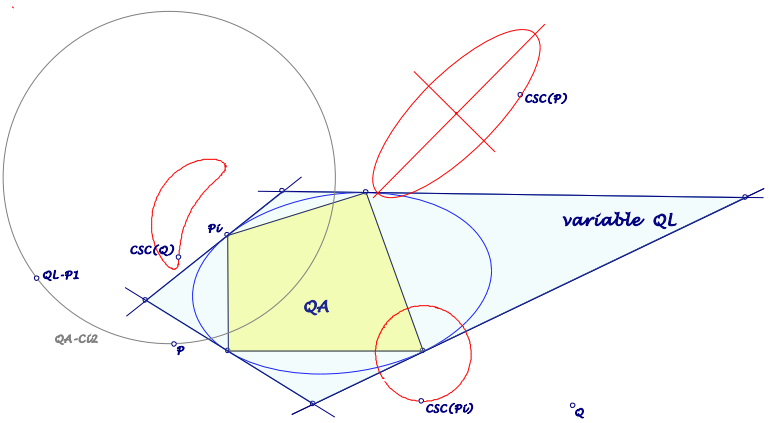
EQF-Note 2017-03-29

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Four Circles for a Quadrangle

Tangential quadrilaterals of a quadrangle and their transformation QL-Tf1 (CSC) lead to a lot of curves. CSC-images of the QA-vertices give four circles (see #2347) and CSC-images of points on QA-Ci2 give special conics.



This is a new aspect, to use the *CSC*-transformation for quadrangle geometry:

Consider a quadrangle *QA* with vertices P_i,
... a variable circumconic *Co*
... with the corresponding tangential quadrilateral *QL*
... and its transformation *CSC*.

- **The reference quadrangle and the tangential quadrilaterals have the same diagonal triangle *DT*.**

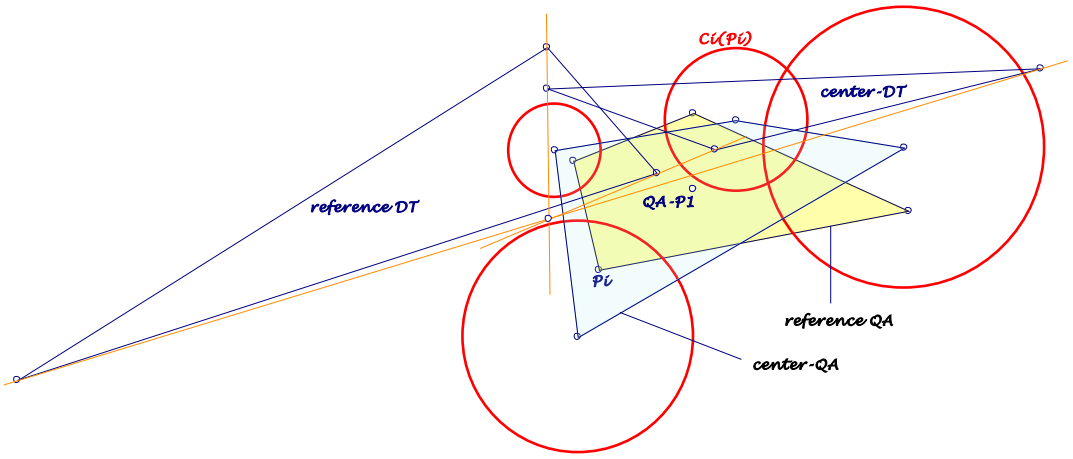
(1) *CSC*-Traces of the *QA*-Vertices

Normally the *CSC*-trace of a point is a quartic. Here two special cases are researched:

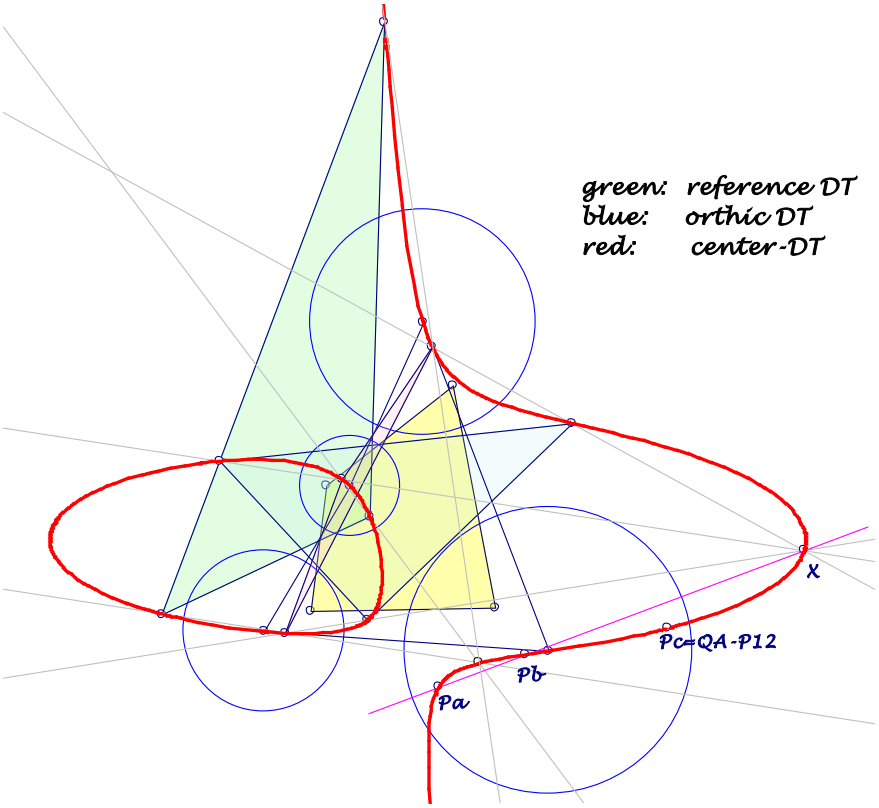
- **The *CSC*-traces of the *QA*-vertices P_i are circles $Ci(P_i)$.**

These four circles are already described in #2347, here a new aspect:

- **The centers of *Ci(Pi)* give a new quadrangle**
  … with a diagonal triangle perspective *DT*
  … and the same *QA-P1*.



- **The centers of the four circles lie on a cubic bearing the following 16 points:**
  … vertices of the center-*QA*,
  … vertices of the reference *DT*,
  … vertices of the orthic triangle of the reference *DT*,
  … vertices of the center-*DT*,
  … *QA-P12* of the reference *QA*,
  … perspector of reference *DT* and center-*DT*,
  … perspector *X* of orthic triangle and center-*DT*.



- **The cubic is a pivotal isocubic …**
  (a) wrt the orthic triangle of the reference *DT*,
  … isoconjugation: isogonal conjugated,
  … pivot P_a: *QA-Tf2* of *X* wrt the center-*QA*.

(b) wrt the center-*DT*,
… isoconjugation: *QA-Tf2* of the center-*QA*,
… pivot P_b: isogonal conjugated of *X* wrt the orthic triangle.
(c) wrt the reference *DT*,
… isoconjugation: swaps *QA-P12* and P_a,
… pivot P_c: *QA-P12*.

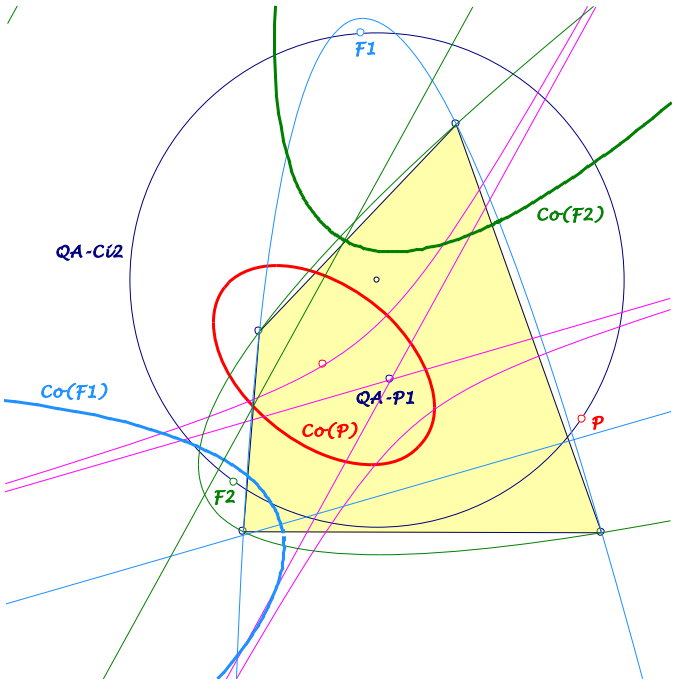
- **The common tangential for the vertices …**
  … of the reference *DT* and *QA-P12* is P_a,
  … of the center-*QA* is P_b,
  … of the orthic triangle and P_a is the isogonal conjugated of P_a wrt the orthic triangle.

(2) *CSC*-Traces of *QA-Ci2*-Points

- **The *CSC*-traces of points *P* on *QA-Ci2* are conics *Co(P)*,**
  … centered on a hyperbola *Hy*
  … with *Hy*-center *QA-P1*.

- **The *CSC*-traces for the vertices of the orthic triangle (on *QA-Ci2*) degenerate collinear on the opposite side.**

- **The *CSC*-traces for the foci of *QA-2Co1* (on *QA-Ci2*, not always real) are parabolas again**
  … with axes parallel to the asymptotes of *Hy*.



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