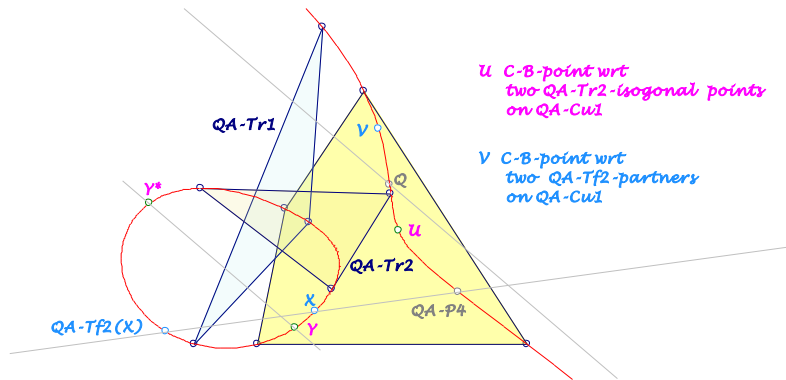


Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Cayley-Bacharach Ninth Point on QA-Cu1

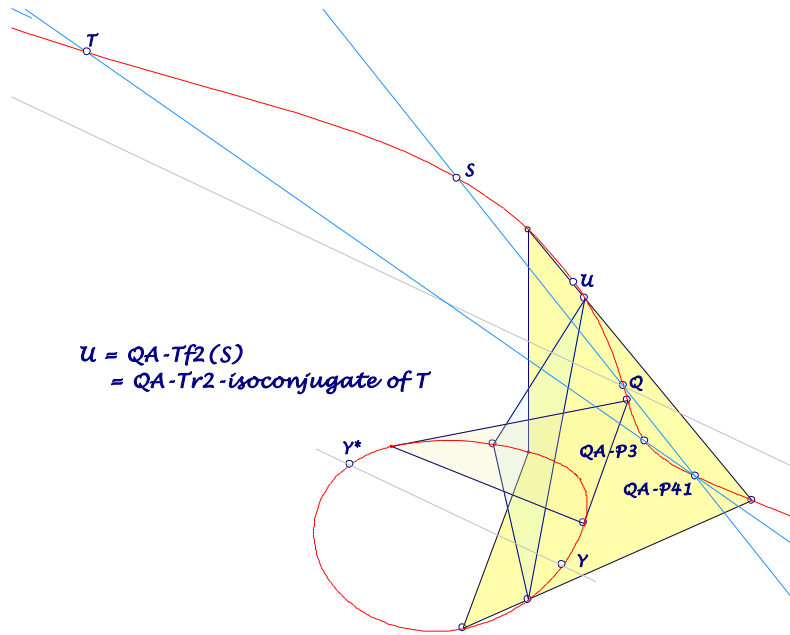
The Cayley-Bacharach ninth point – here shortened C-B-point – is the ninth common point of cubics through eight given points. A.S. Hart gives a construction of the C-B-point in [1]; a mistake is corrected in QFG-message 2447. – With a CABRI-macro here are researched the C-B-points for the vertices of the diagonal triangle QA-Tr1, the vertices of the Miquel triangle QA-Tr2 and two points on the cubic QA-Cu1, which are QA-Tr2-isogonal conjugated or QA-Tf2-partner.



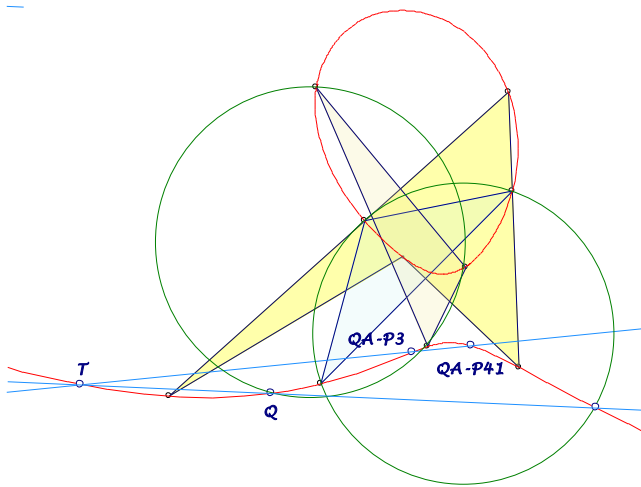
The cubic QA-Cu1 is invariant to the isogonal conjugation wrt the Miquel triangle QA-Tr2 and the transformation QA-Tf2. Let Q be the intersection of QA-Cu1 and its asymptote. Two QA-Tr2-isogonal conjugated points on QA-Cu1 lie on a parallel to the asymptote. Two QA-Tf2-partners on QA-Cu1 are collinear with QA-P4.

(1) Two QA-Tr2-isogonal points on QA-Cu1

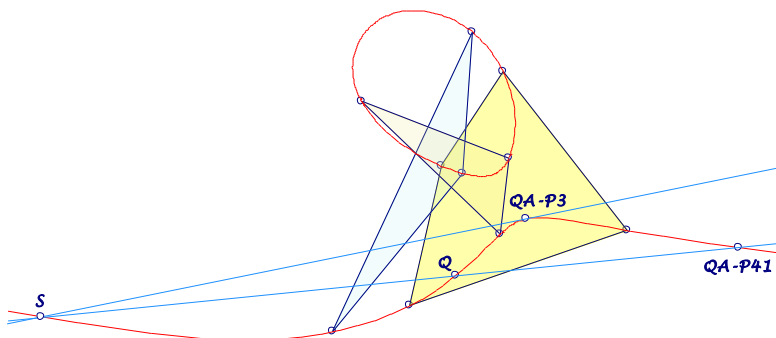
- The Cayley-Bacharach point of
 ... the vertices of QA-Tr1 and QA-Tr2
 ... and two QA-Tr2-isogonal points on QA-Cu1
 ... is a fixed point U on the cubic QA-Cu1.
- The C-B-point U is
 ... the QA-Tr2-isogonal conjugated of the third
 intersection T of QA-Cu1 and QA-P3.QA-P41,
 as well as
 ... the QA-Tf2-image of the third intersection S of
 QA-Cu1 and Q.QA-P41.



- The point T on $QA-Cu1$ is the intersection ... of $QA-P3.QA-P41$ and the line, ... connecting the fourth intersections of $QA-Cu1$ and the circumcircles of $QA-Tr1$ and $QA-Tr2$.

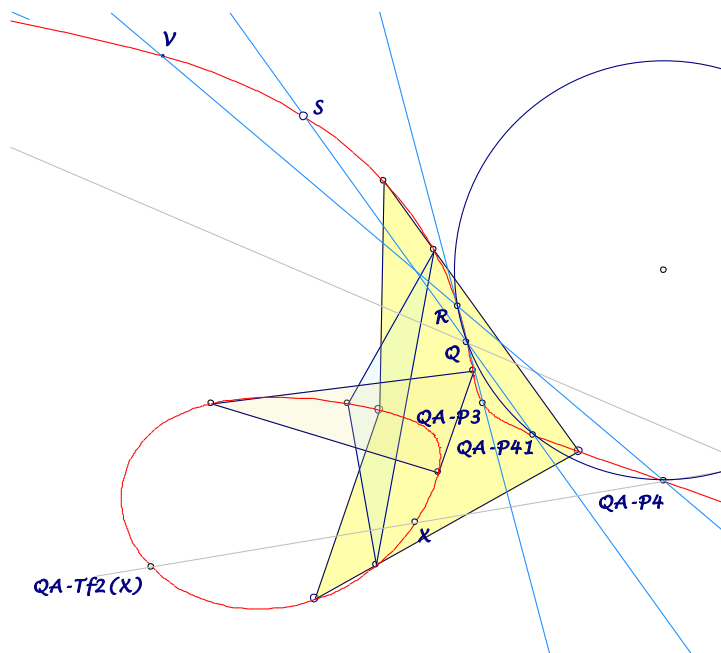


- The point S on $QA-Cu1$ is the intersection ... of $Q.QA-P41$ and the tangent in $QA-P3$ at $QA-Cu1$.



(2) Two QA-Tf2-partners on QA-Cu1

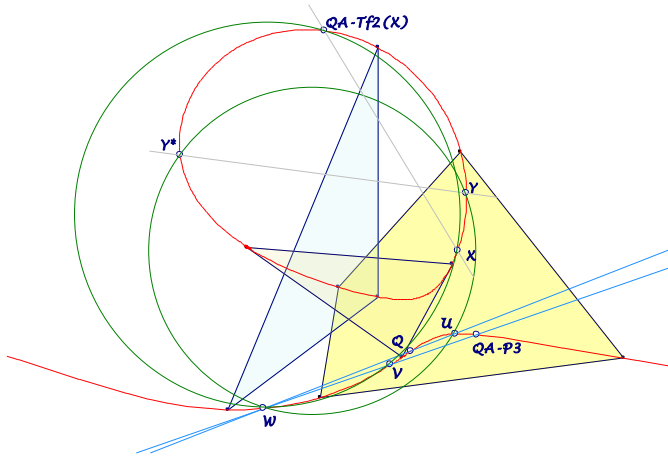
- The Cayley-Bacharach point of
 ... the vertices of $QA-Tr1$ and $QA-Tr2$
 ... and two $QA-Tf2$ -partners on $QA-Cu1$
 ... is a fixed point V on the cubic $QA-Cu1$.
- The $C-B$ -point V is
 ... the $QA-Tr2$ -isogonal conjugated of the third
 intersection S (see above) of $QA-Cu1$ and $Q.QA-P41$,
 as well as
 ... the $QA-Tf2$ -image of the third intersection R of
 $QA-Cu1$ and $Q.QA-P3$.



- The point R is the second intersection
 ... of the line $Q.QA-P3$
 ... and the circumcircle of $Q, QA-P4, QA-P41$.

Finally:

- Circumcircles
 ... of two $QA-Tr2$ -isogonal points on $QA-Cu1$ and U
 as well as
 ... of two $QA-Tf2$ -partner on $QA-Cu1$ and V
 ... have a fixed fourth intersection W on $QA-Cu1$.
- The point W is the intersection of $U.Q$ and $V.QA-P3$.



[1] A. S. Hart: Construction by Ruler alone to determine the ninth Point of Intersection of two Curves of the third degree. Cambridge and Dublin Mathematical Journal 6 (1851) 181-182.

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