EQF-Note 2017-06-12

Cayley-Bacharach Ninth Point on QA-Cu1 (II)

*This note is a direct continuation of EQF-Note 2017-
06-02 in QFG-message 2458, where Cayley-Bacharach
ninth points are discussed wrt QA-Tr1, QA-Tr2 and two
points on QA-Cu1. Here finally also C-B-points of the
reference QA and four points on QA-Cu1 are
researched.*
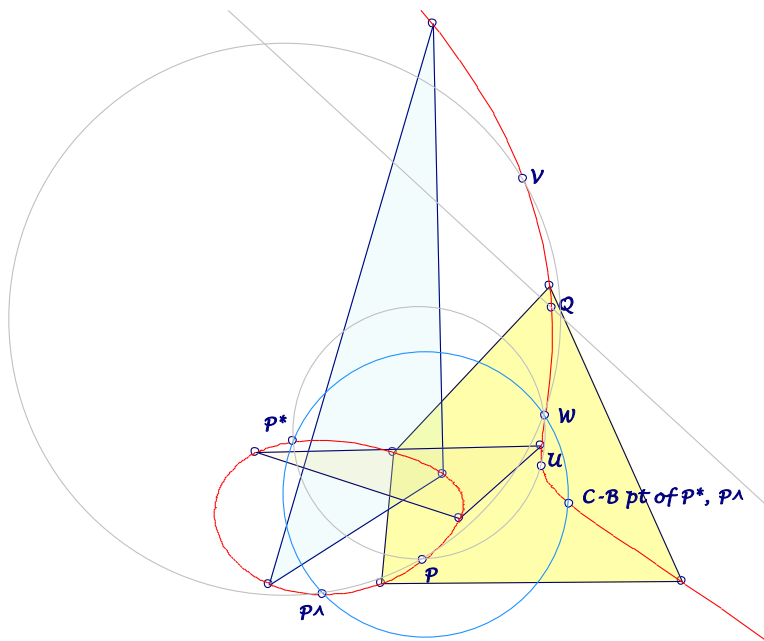
Remember (1) and (2) of the previous note:

(1)   Two QA-Tr2-isogonal points on QA-Cu1

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tr2*-isogonal points on *QA-Cu1*
  … is a fixed point *U* on the cubic *QA-Cu1*.

- **The *C-B*-point *U* is**
  **… the *QA-Tr2*-isogonal conjugated of the third
  intersection *T* of *QA-Cu1* and *QA-P3.QA-P41*,**
                     as well as
  **… the *QA-Tf2*-image of the third intersection *S* of
  QA-Cu1 and *Q.QA-P41*.**

(2)   **Two QA-Tf2-partners on QA-Cu1**

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tf2*-partners on *QA-Cu1*
  … is a fixed point *V* on the cubic *QA-Cu1*.

- **The *C-B*-point *V* is**
  … the *QA-Tr2*-isogonal conjugated of the third intersection *S* (see above) of *QA-Cu1* and *Q.QA-P41*,
                    as well as
  … the *QA-Tf2*-image of the third intersection *R* of *QA-Cu1* and *Q.QA-P3*.

- **Circumcircles**
  … of two *QA-Tr2*-isogonal points on *QA-Cu1* and *U*
                    as well as
  … of two *QA-Tf2*-partner on *QA-Cu1* and *V*
  … have a fixed fourth intersection *W* on *QA-Cu1*.

- **The point *W* is the intersection of *U.Q* and *V.QA-P3*.**

New:

**(3)**   **Two or Four points on *QA-Cu1***

Nominations: Let Q be the intersection of the cubic *QA-Cu1* and its asymptote. Let P^* be the isogonal conjugate of P wrt the Miquel triangle *QA-Tr2* and $P^{\wedge}$ the *QA-Tf2*-image of P. For P on *QA-Cu1* these points P^* and $P^{\wedge}$ are also on *QA-Cu1*. For two points X, Y on *QA-Cu1* let $res(X,Y)$ be the third intersection of *QA-Cu1* and *XY*. Finally: X' shall be the tangential of X.

The properties above can be generalized:

> **The *C-B*-point wrt *QA-Tr1* and *QA-Tr2***
> **… and two points *X, Y* on *QA-Cu1***
> **… is the fourth intersection of *QA-Cu1* and the circumcircle of *X, Y, W*.**

- **Points *X, Y* on *QA-Cu1***
  … with the same 3rd intersection *P* of *XY* and *QA-Cu1*
  … have the same *C-B*-point wrt *QA-Tr1* and *QA-Tr2*,
  … which is the 3rd intersection of *QA-Cu1* and *WP**.

> **The *C-B*-point wrt *QA-Tr1* and *QA-Tr2***
> **… of *X, Y* on *QA-Cu1***
> **is the *C-B*-point wrt the reference *QA***
> **… of *P = res(X,Y)*, *P**, *P^* and tangential *U´* of *U*.**

CB
≈ C-B-point of X, Y
   wrt QA-Tr1, QA-Tr2
≈ C-B-point of P, P*, P^, U'
   wrt QA
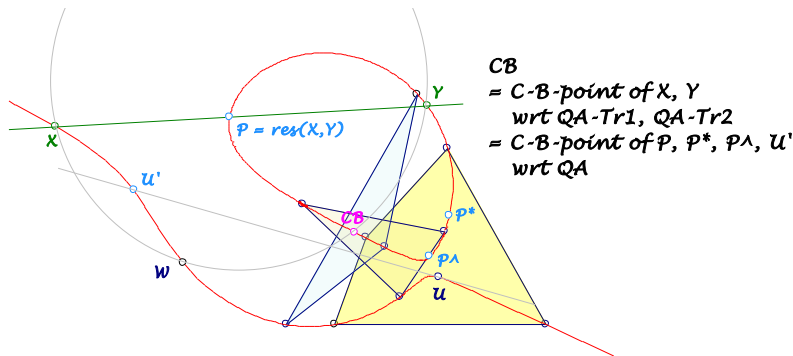
Final remarks

The results lead to new points *U, V*, their tangentials *U´, V´* and *W* on *QA-Cu1*. Point *W* can be taken as basic point in the following sense:

- $U = res(W,Q)$,
  $V = res(W, QA\text{-}P3)$,
  $U´ = res(W^\wedge, QA\text{-}P41)$,
  $V´ = res(W^*, QA\text{-}P3)$.

> **The *C-B*-point wrt *QA* of *U, V, U´, V´* is *W**.**

Further *C-B*-points:

- **The *C-B*-point wrt *QA-Tr1, QA-Tr2* of *U, V* is *V**.**
- **The *C-B*-point wrt *QA-Tr1, QA-Tr2* of *W, W** is *U*.**
- **The *C-B*-point wrt *QA-Tr1, QA-Tr2* of *W, W^* is *V*.**
- **The *C-B*-point wrt *QA-Tr1, QA-Tr2* of *W*, W^* is *V´*.**

- **Four points on *QA-Cu1* with $res(A,B) = res(C,D) = P$**
  … have *C-B*-point wrt *QA*
  … in $res(P´,QA\text{-}P41)$.

> **A perspective quadrangle on *QA-Cu1***
> **… with perspector *P* on *QA-Cu1***
> **… has a *C-B*-point wrt the reference *QA***
> **… in the tangential of the tangential of *P*.**

- **The *C-B*-point of the *QA*-vertices and their *QA-Tr2*-isogonal conjugates**
  … is the tangential of *Q*.

> **The *C-B*-point of two quadrangles on *QA-Cu1* with the same Miquel triangle**
> **… is the third intersection of *QA-Cu1* and the line through their *QA-P41*-points.**

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