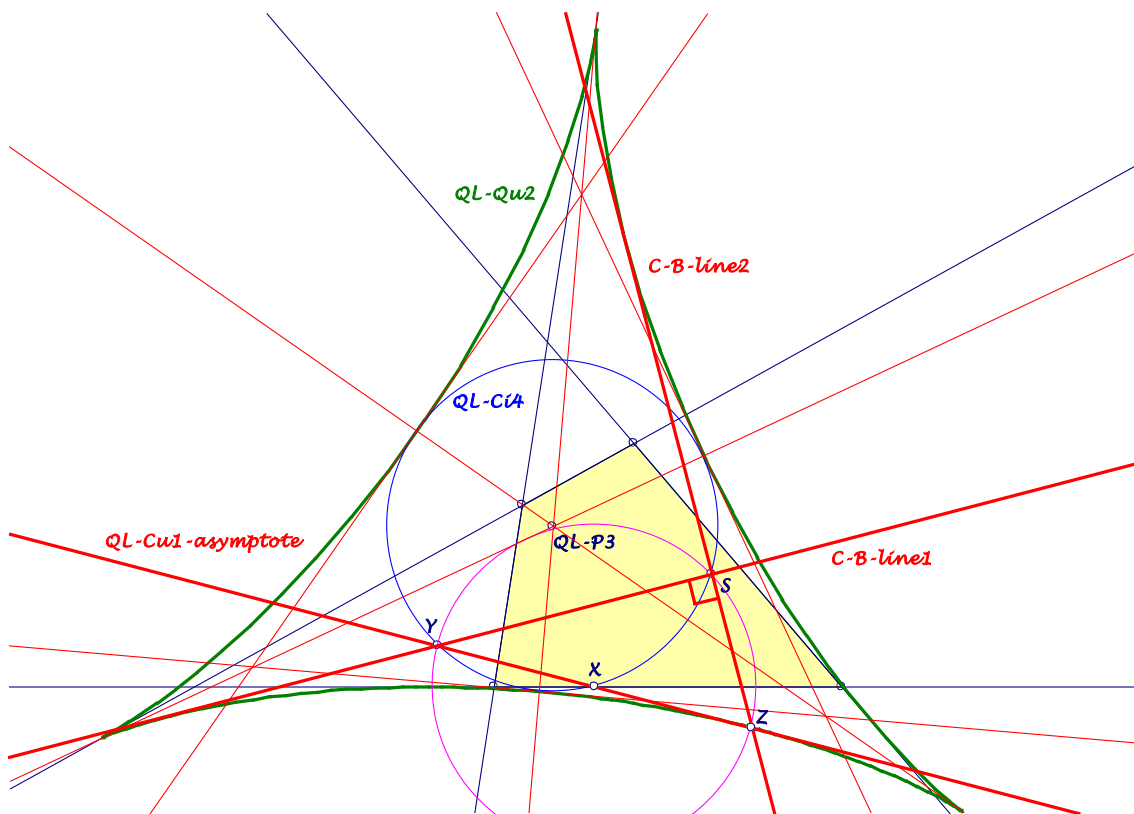


Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Cayley-Bacharach Lines for QL-Qu2

Two orthogonal ninth Cayley-Bacharach tangents for the deltoid QL-Qu2 of a quadrilateral are researched, considering the four lines of the quadrilateral, the asymptote of QL-Cu1 and the three common tangents of QL-Qu2 and QL-Ci4 on the one hand or the three axes of symmetry for QL-Qu2 on the other hand.



In *QFG*-message 2511 a Cayley-Bacharach ninth line is defined:
Consider eight lines,
... four lines for a reference QL and its dual QA ,
... duality gives eight points for the eight lines,
... take their Cayley-Bacharach ninth point,
... its dual will give the Cayley-Bacharach ninth line.
For duality see *EQF*-message 1516, a construction for the Cayley-Bacharach ninth point is described in *EQF*-message 2471.

The deltoid $QL-Qu2$ is a quartic of class three, so eight tangents will determine a ninth Cayley-Bacharach tangent.

There are the following well known tangents of $QL-Qu2$:

1. four QL -lines,
2. the $QL-Cu1$ -asymptote,
3. three common tangents of $QL-Qu2$ and $QL-Ci4$,
4. three axes of symmetry for $QL-Qu2$.

The lines 1, 2, 3 will give the $C-B$ -line 1, the lines 1, 2, 4 will give the $C-B$ -line 2.

- **The $C-B$ -lines 1, 2 are orthogonal tangents at $QL-Qu2$
...with intersection S on $QL-Ci4$.**

Alternatively the two lines can be constructed as follows:

- ... The $QL-Cu1$ -asymptote cuts $QL-Ci4$ in the points X and Y ,
- ... X shall be the middle of the asymptote as chord of $QL-Qu2$.
- ... A circle Ci round X through Y
- ... cuts the asymptote once more in Z ,
- ... cuts the circle $QL-Ci4$ once more in S .
- ... S is the intersection of the $C-B$ -lines,
- ... Y and Z their intersections with the asymptote.

Further properties of the configuration:

Let Tr be the equilateral triangle of the contact points of $QL-Qu2$ and $QL-Ci4$:

The Simson line of X wrt Tr

- ... is orthogonal to the asymptote
- ... through the middle of $X.QL-P3$.

The Simson line of Y wrt Tr

- ... is parallel $X.QL-P3$,
- ... half of the distance
- ... is parallel $C-B$ -line 2,
- ... fourth part of the distance.

The Simson line of S wrt Tr

- ... is parallel $Y.QL-P3$
- ... half of the distance.

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