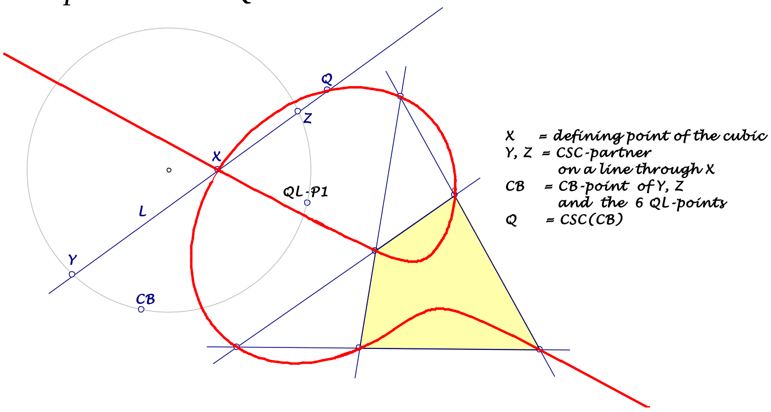
EQF-Note 2017-09-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

New QL-circumscribed Cubics II

In EQF-Note 2017-09-22 new QL-circumscribed cubics were described using the Cayley-Bacharach ninth point. Here alternative a simpler construction is offered and further properties are mentioned, if the defining point lies on QL-Cu1.



Alternative construction

There is an interesting property in dual geometry. The following two constructions give the same point:

(1) For a line *L*
… the ***QA-Tf2***-partner on *L*
… and their *DT*-cevian triangles
… give a Cayley-Bacharach ninth point *Q* on *L*.

(2) For a line *L*
… the ***QL-Tf1***-partner on *L*
… and the six *QL*-points
… give a Cayley-Bacharach ninth point *CB*,
… with *QL-Tf1(CB) = Q* on *L*.

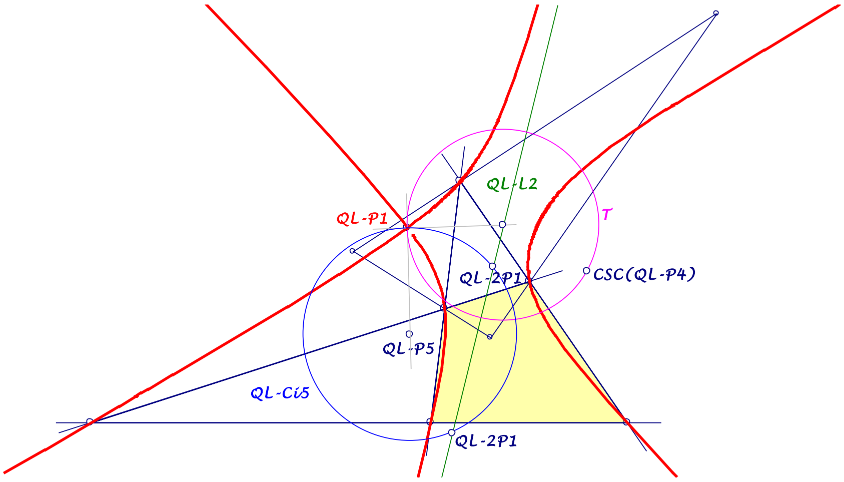
The construction of the *CB*-point in (2) is comparatively simple.

The points *Q* for a line pencil wrt a defining point *X* give a *QL*-circumscribed cubic *QL-Cux*, which can be described as nonpivotal isocubic (see *QFG*-message 2610):
… reference triangle ***Tr*** is the cevian triangle of *X* wrt the *QL*-diagonal triangle,
… isoconjugation ^ with fixed point *X*
… and Bernard Gibert´s circle *Γ* (*EQF*: *Ref [17b], 1.5.6*).
Then *QL-Cux* is the locus of points *M* such that *M* and *M*^ are conjugated wrt the circle *Γ*.

QL-Cux* with *X = QL-P1:

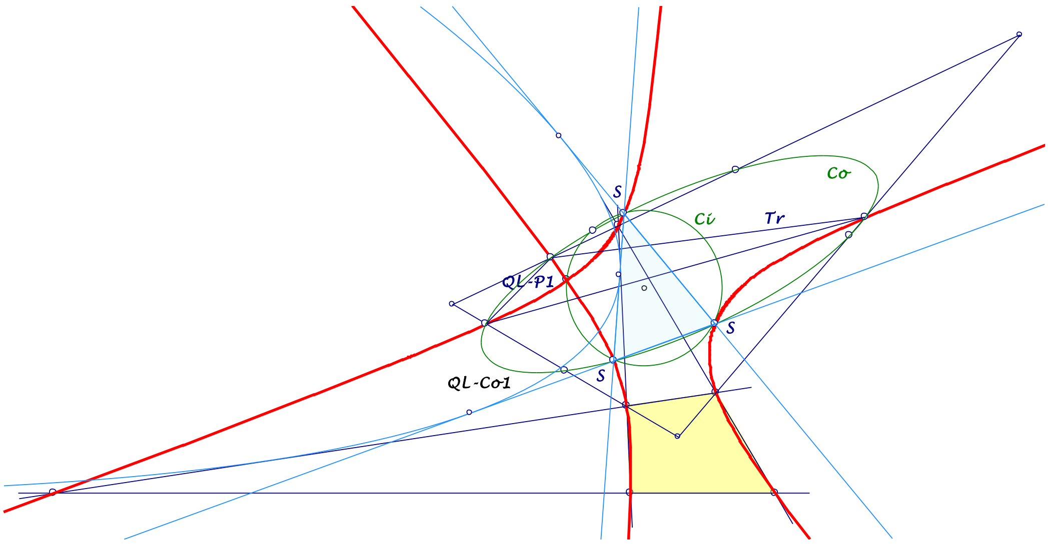
A construction of this cubic is only possible with method (1).

- **The defining point *QL-P1* is double point of the cubic with orthogonal tangents, which are the Steiner axes.**



- **The circle *Γ* for *X = QL-P1***
  … bears *QL-P1* and *CSC(QL-P4)*,
  … is centered on *QL-L2*,
  … tangent to *QL-P1.QL-P5*,
  … orthogonal to *QL-Ci5*,
  … with inverse Plücker points *QL-2P1*.

Two helpful curves:
… conic ***Co*** circumscribed the cevian triangle *Tr* of *QL-P1* through the midpoints of the diagonal triangle (^isoconjugate of the line at infinity),
… circle ***Ci*** through *QL-P1*, which is the *CSC*-image of a *QL-L1*-parallel through *QL-P21*.
One of the four intersections of *Co* and *Ci* lies on the circumcircle of *Tr*.

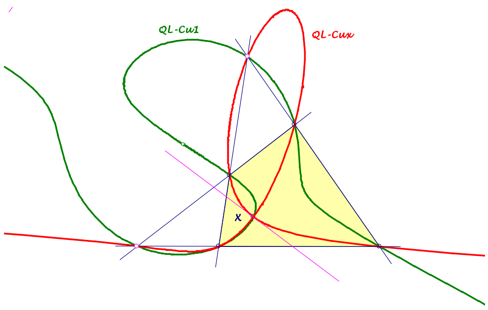


- **Three intersections *S* of *Co* and *Ci* lie on the cubic *QL-Cux*:**
  … their ^isoconjugates are the points at infinity of the asymptotes,
  … the sidelines of their triangle are parallels to the asymptotes
  … and tangent to the inscribed parabola *QL-Co1*.

Cubics *QL-Cux* with *X* ≠ *QL-P1* on *QL-Cu1:*

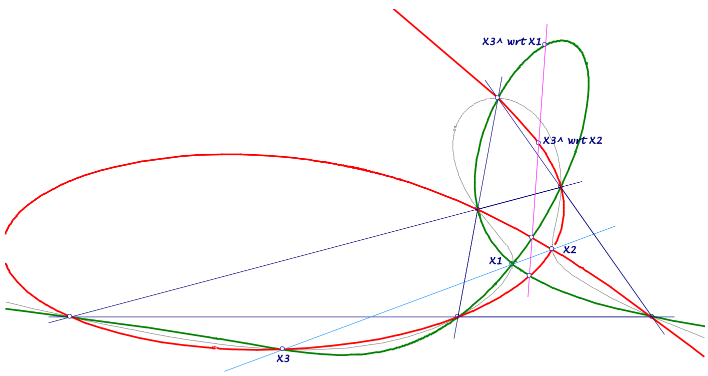
- **For *X* on *QL-Cu1*, the cubic *QL-Cux* bears always the defining point *X*.**

- **The defining point *X* as double point has orthogonal *QL-Cux*-tangents,**
  … which are the angle bisectors at *X* wrt two *CSC*-partner on *QL-Cu1*.



For *X* in an intersection of *QL-Cu1* and *QL-L1* these tangents are the axes of *QL*-inscribed conics. In general: For *X* on *QL-Cu1* these orthogonal tangents are the degenerated polar conics of *QL-Cu2*.

- **The cubics *QL-Cux* for *CSC*-partner X_1 and X_2 on *QL-Cu1* intersect**
  … in the 3rd intersection X_3 of X_1X_2 and *QL-Cu1*
  … and two other points
  … collinear with the ^isoconjugated of X_3 wrt X_1, X_2.



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