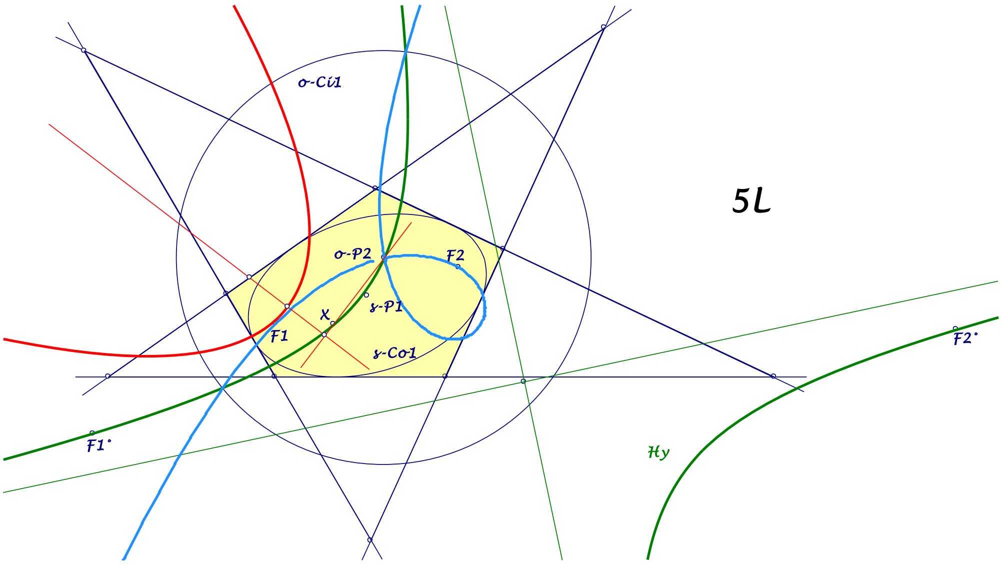
EQF-Note 2017-10-25

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Polygon Geometry
http://www.chrisvantienhoven.nl/

Orthogonal 5L-Hyperbola

This hyperbola is already mentioned in QFG#762, 769, 780, 790, also in EPG under 5L-s-Tf1. Wrt special transformations it leads to a 5L-parabola.



Starting with the *5L*-inscribed conic *5L-s-Co1* and Clifford´s circle *5L-o-Ci1*, we get an interesting point

$$X = F_1F_2° \cap F_1°F_2.$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
$F_1°$ and $F_2°$ inverses of F_1 and F_2 wrt *5L-o-Ci1*.

The transformation *5L-s-Tf1* maps *X* to the center of Clifford´s circle *5L-o-P2*, but this transformation isn´t reciprocal.

- **The intersections**
  … of lines through *X* and their *5L-s-Tf1*-image lines
  … give an orthogonal hyperbola *Hy* (*QFG#762*).

- **The orthogonal hyperbola *Hy***
  … is centered in the midpoint of $F_1°F_2°$,
  … bears $F_1°$, $F_2°$, *5L-o-P2*, *X*
  … and the fixed points of *5L-s-Tf1*,
  … is tangent to *5L-s-P1.5L-o-P2* and *5L-s-P1.X*.
  … Polars of F_1,F_2 intersect in $F_1°.F_2° \cap X.5L\text{-}o\text{-}P2$.

In *QFG#780* there is a transformation *Tf2*,
... that maps a line *L* (or circle *Ci*) to the common point
... of all radical axes wrt the 5 *CSC*-circles of *L* (or *Ci*).

- **The *Tf2*-image of the line pencil of *X* is a strophoid,**
  … for a line connecting *5L-o-P2* and the *Hy*-center,
  … fixed point *5L-o-P2*
  … and pole in the *5L-o-Ci1*-inverse of the reflection of *5L-o-P2* in the center *5L-s-P1* of the inscribed conic.

- **The strophoid is the inverse of the hyperbola *Hy* wrt *5L-o-Ci1*.**

In *QFG#790* there is are reciprocal transformations *Tf3* and *Tf4*:
... *Tf3* maps a point to the radical axis of its *CSC*-circle and *5L-o-Ci1*,
… *Tf4* maps a line to the common point of the radical axes of the *CSC*-circles of its points.

- ***Tf4* maps the *Hy*-tangents to a parabola**
  … with directrix *X.5L-o-P2*
  … and focus in *Tf4* of the perpendicular line of *Z.5L-o-P2* in *Z* (*Z Hy*-center).

Then *Hy* is the envelope of the *Tf3*-image lines of points on this parabola.

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