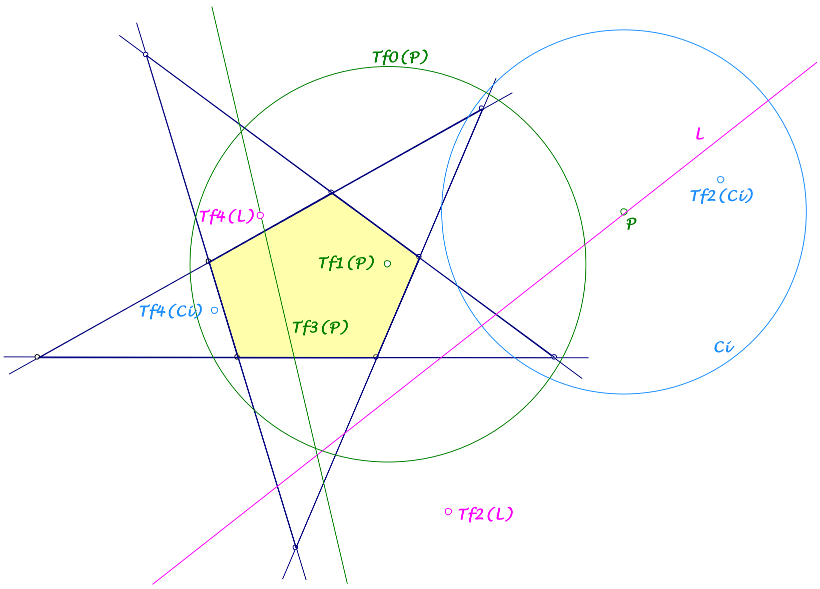
EQF-Note 2017-11-04

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-related 5L-Transformation II

Wrt the CSC-Transformation QL-Tf1 for quadrilaterals several transformations for 5-lines can be considered.



Transformations

Tf0 ***point → circle***: The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).

Tf2 ***line/circle → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line/circle → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Inverse Transformations

$Tf0_{inv}$ *Tf0(P)* are special circles, *Tf2* and *Tf4* of these circles give *P* again.

$Tf1_{inv}$ … does not exist. Every point has two pre-images, which are partner wrt the transformation *Tf* in *QFG# 2669*.

$Tf2_{inv}$ … does not exist. *Tf2(L)* is a point, which has three pre-images. Let *Tf2(Lx)* = *Tf2(Ly)* = *Tf2(Lz)*, then

Tf4(Lx)=Ly∩Lz, Tf4(Ly)=Lx∩Lz, Tf4(Lz)=Lx∩Ly.
$Tf3_{inv}$ *Tf3(P)* is a line and *Tf4* of this line is *P* again.
$Tf4_{inv}$ *Tf4(L)* is a point and *Tf3* of this point is *L* again,
Tf4(Ci) is a point and *Tf0* of this point is *Ci* again.

Tf-Geometry for *5L-o-Ci1* and *5L-s-Co1*

For properties of the transformations see the cited messages. Here finally the *Tf*-geometry of the Clifford circle *5L-o-Ci1* and the inscribed conic *5L-s-Co1* shall be researched.

- **For points *X* on *5L-o-Ci1* the degenerated circles *Tf0(X) = Tf3(X)* are tangents of *5L-s-Co1*.**

- **For points *X* on *5L-o-Ci1* the points *Tf1(X)* are points at infinity with direction orthogonal *Tf3(X)*.**

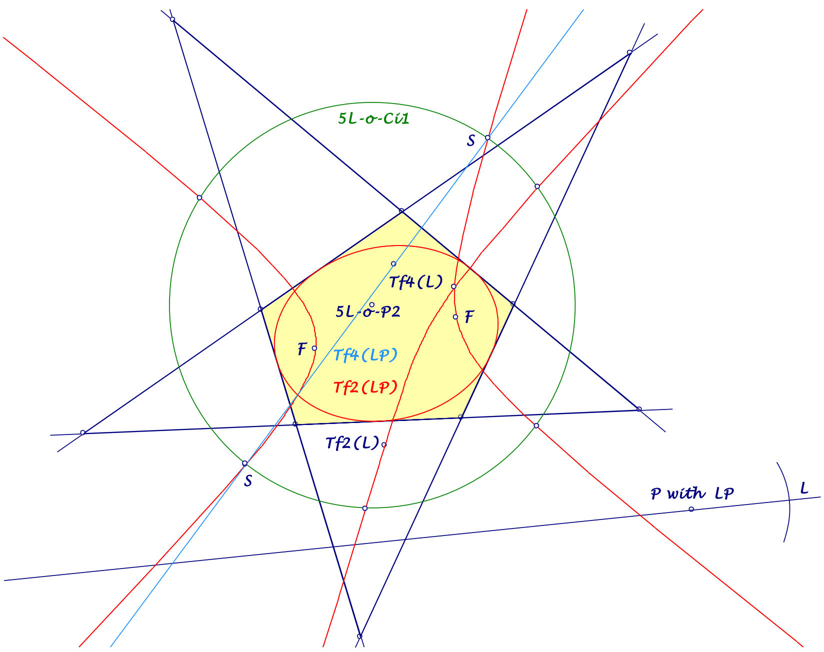
- **For points *X* on *5L-s-Co1* the circles *Tf0(X)* contact *5L-o-Ci1*.**

- **For tangents *L* at *5L-o-Ci1* the points *Tf4(L)* lie on *5L-s-Co1*.**

- **For tangents *L* at *5L-s-Co1* the points *Tf2(L) = Tf4(L)* lie on *5L-o-Ci1*.**

- **For points *X* on a line *L* the points *Tf1(X)* lie on a conic through *5L-o-P2*.**

- **For points *X* on a line *L* the lines *Tf3(X)* give a line pencil of *Tf4(L)*.**



- ***Tf2* maps a line pencil *LP* to a cubic**
  … through the foci F_1, F_2 of *5L-s-Co1*
  … with $Tf2(PF_i) = F_j$.

- ***Tf4* maps a line pencil *LP* to a line,**
  … intersecting *5L-o-Ci* on the cubic in S_1, S_2
  … with $Tf0(S_i)$ tangents from *P* at *5L-s-Co1*,
  … … $Tf3(S_i) = PS_j$ tangent at *5L-s-Co1*.
  … … $Tf2(S_1S_2)$ is a double point of the cubic,
  … … $Tf4(S_1S_2)$ is the point *P* again,
  … … $Tf2(Tf0(S_i)) = Tf4(Tf0(S_i)) = S_j$.

- **The *Tf2*-cubic of a line pencil *PL* intersects *5L-o-Ci* in S_1, S_2 and four further points *T* with *T* on *Tf2(PT)*.**

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