

EQF-Note 2017-11-06

Background for these notes is:

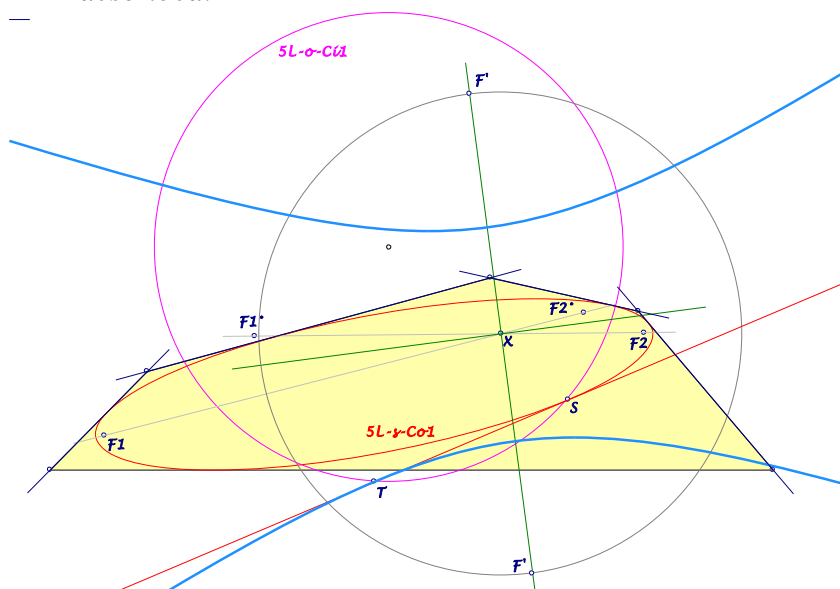
Chris van Tienhoven:

Encyclopedia of Quadri-Figures and Poly Geometry

<http://www.chrisvantienhoven.nl/>

5L-Example for Polarity with Conic

“According to Von Staudt, a conic corresponds to a polarity”: This is discussed in *QFG*, started by Tsihong Lau *QFG*#2680. In *QFG*#2683 there are two 5L-transformations $Tf_3(P)$, $Tf_4(L)$, which define a polarity. Here a construction of the corresponding conic is described.



If we have a look in *QFG*#2683 and take the transformation ...

Tf_0 *point* \rightarrow *circle*: The 5 CSC-images of a point P wrt the 4L of a 5L are concyclic on the circle $Tf_0(P)$.

... we get two further transformations

Tf_3 *point* \rightarrow *line*: $Tf_3(P)$ is the radical axis of $Tf_0(P)$ and 5L-o-Ci1 (see *QFG*#790).

Tf_4 *line* \rightarrow *point*: Common point of the radical axes for the Tf_0 -circles of the points of the line (see *QFG*#790).

... which give a bijection between points and lines, that preserves the incidence relation. The corresponding conic can be constructed as follows:

- Center of the conic is the point X (see *QFG#2669*)

$$X = F_1F_2^\bullet \cap F_1^\bullet F_2.$$
 F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
 F_1° and F_2° inverses of F_1 and F_2 wrt *5L-o-Cil*.
- Main axis is the angle bisector of $\angle F_1XF_2$.
- Foci F' are the intersections of the main axis and a circle round X with radius $\sqrt{X.F_1 \times X.F_2}$.
- Special points T of the hyperbola (not always real):
 ... Let S be an intersection of *5L-s-Co1* and *5L-o-Cil*,
 ... Tg the tangent in S at *5L-s-Co1*,
 ... T second intersection of Tg and *5L-o-Cil*.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de