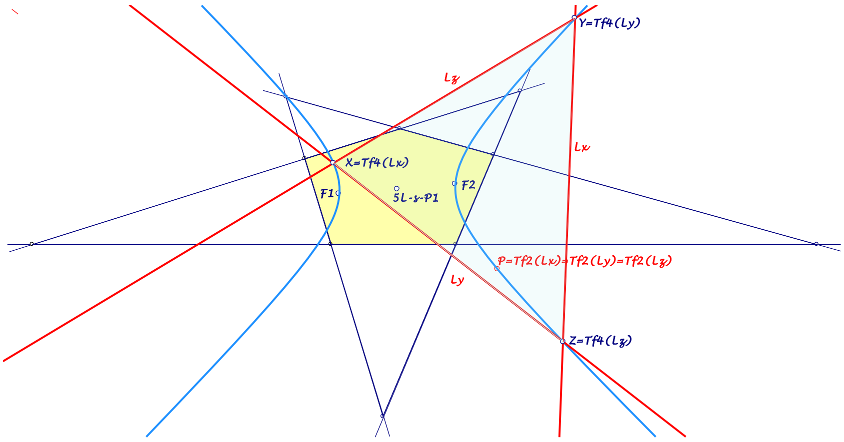
EQF-Note 2017-11-15

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-related *5L*-Transformation III

The 5L-transformation Tf2 in QFG#2683 maps a line to a point, but there are two other lines with the same image. Here a construction of these further lines is given.



In *QFG#2683* several *5L*-transformations are described:

- ***Tf0*** ***point*** → ***circle:*** The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.
- ***Tf1*** ***point*** → ***point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).
- ***Tf2*** ***line*** → ***point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).
- ***Tf3*** ***point*** → ***line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).
- ***Tf4*** ***line*** → ***point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Tf3 and *Tf4* are inverse transformations. But wrt *Tf1* every point has two pre-images, which are partner wrt the transformation *Tf* in *QFG#2669*. *Tf2* has the curious property, that there are three lines with the same image point.

For a line L_x and its image point $Tf2(L_x)$ there are two other lines L_y, L_z with $Tf2(L_x) = Tf2(L_y) = Tf2(L_z)$.

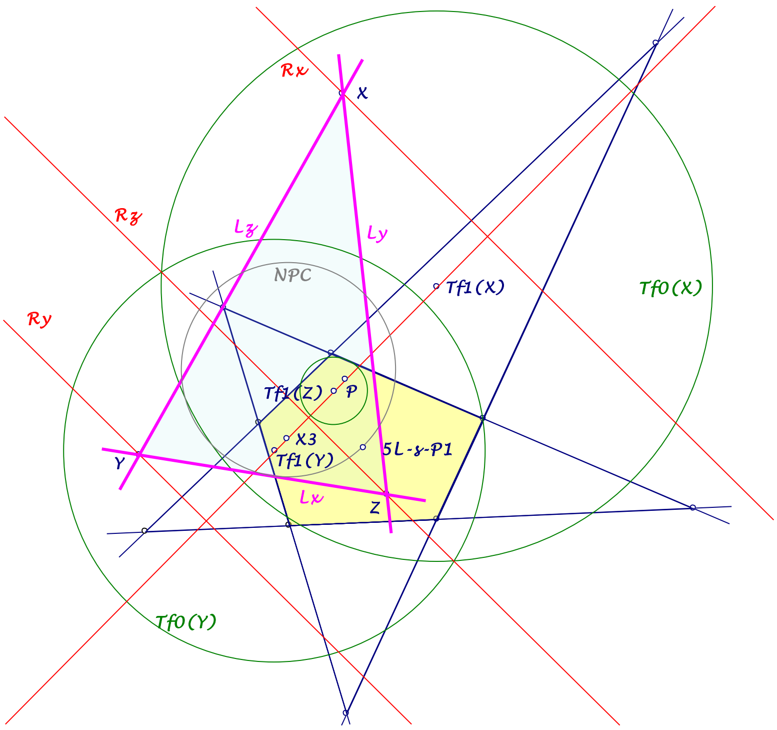
Construction:

We start with a line L_x,
… its *Tf2*-point P, its *Tf4*-point X and the foci F_1,F_2 of the inscribed conic *5L-s-Co1*
… and construct the orthogonal hyperbola *Hy*,
… … centered in *5L-s-P1*,
… … through P, X, F_1, F_2.
… The intersections Y and Z of L_x and *Hy*
… give the lines $L_y = XZ$ and $L_z = XY$.

Properties:

- **For points X, Y, Z with $L_x = YZ$, $L_y = ZX$, $L_z = XY$ … and $Tf2(L_x) = Tf2(L_y) = Tf2(L_z)$ holds:**
  $$Tf4(L_x) = X,\ Tf4(L_y) = Y,\ Tf4(L_z) = Z,$$
  $$Tf3(X) = L_x,\ Tf3(Y) = L_y,\ Tf3(Z) = L_z.$$

- **The points X, Y, Z, F_1, F_2 and $P = Tf2(L_{x,y,z})$ lie on an orthogonal hyperbola, centered in *5L-s-P1*.**



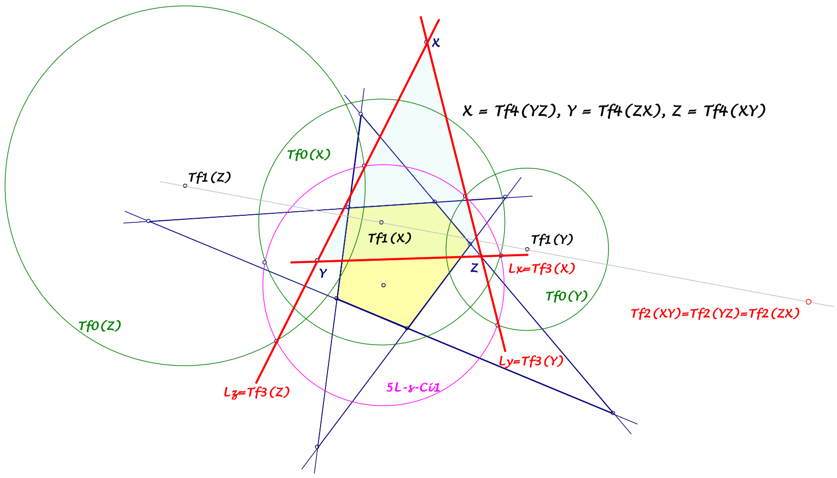
- **The nine-point circle of *XYZ* bears *5L-s-P1*.**

- **The *Tf1*-images of X, Y, Z are collinear with $P = Tf2(L_{x,y,z})$ and the orthocenter *X3* of *XYZ*.**

- **The radical axis R_z of the circles *Tf0(X)* and *Tf0(Y)* bears Z and is parallel to R_x and R_y …**

- **The radical axis of *Tf0(X)* and *5L-s-Ci1* is *YZ*, …**

Finally a figure with all transformations for a triangle *XYZ* with
Tf2(YZ) = Tf2(ZX) = Tf2(XY).



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