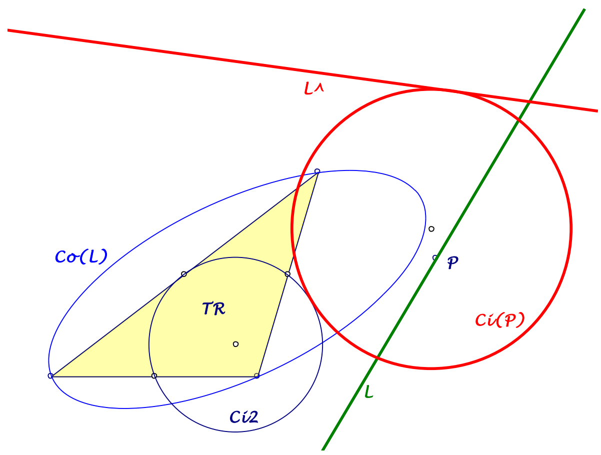
EQF-Note 2018-01-13

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Triangle Circles and 5P-s-Tf3

The transformation 5P-s-Tf3 depends only on the circumconic of the 5P (see QFG#2811). So this transformation can be used for triangles wrt a chosen conic. Here a related line-line transformation is researched wrt a conic depending on the line to be transformed.



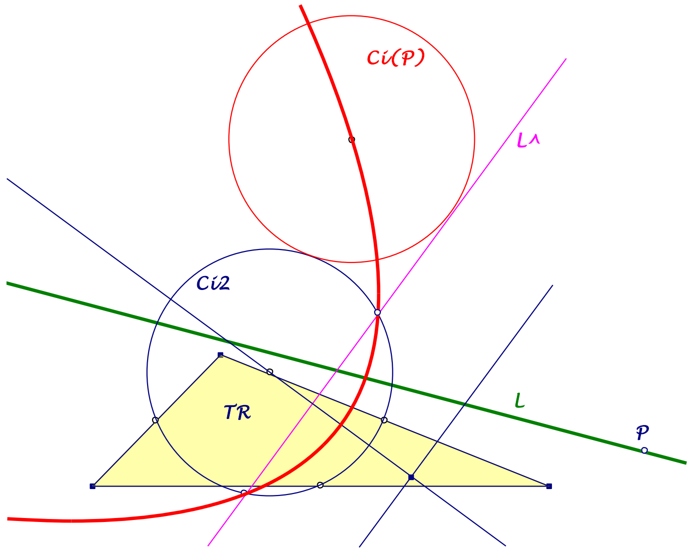
We start with a triangle *TR* and a line *L*,
… the *TR*-isogonal conjugate of *L* gives a circumscribed conic *Co(L)* of *TR*.
… *5P-s-Tf3* wrt the conic *Co(L)* maps *L* to another line *L^*.
… The lines *L^* for the line pencil of a point *P*
… … envelope a circle *Ci(P)*.

We study the transformations $L \to L^\wedge$ and $P \to Ci(P)$:
Let Ci_1 be the circumcircle and Ci_2 the nine-point circle of *TR*.

- **Circles *Ci(P)* contact Ci_2.**

- **For points *P* on circle Ci_1**
  … circles *Ci(P)* degenerate in points on Ci_2.

- **Circles *Ci(P)* for points *P* on a line *L***
  … contact *L^* and Ci_2,
  … have centers on a parabola *Pb(L)*
  … … with focus in the center of Ci_2,
  … … bearing the intersections of Ci_2 and *L^*,
  … … directrix parallel *L^*.



- **For *P* as in-/excenter of *TR***
  … circle *Ci(P)* is the corresponding in-/excircle of *TR*.

- **For *L* as *TR*-sideline**
  … line *L*^ is a sideline of the orthic triangle of *TR*.

Final remarks

If we replace the isogonality in the starting procedure by another isoconjugation, we will not get circles *Ci(P)* but conics *Co(P)*. For the fixed points *F* of the isoconjugation the conics *Co(F)* are inscribed *TR* and have one focus *F*.
Consequently for quadrangles $P_1P_2P_3P_4$ with diagonal triangle *QA-Tr1* = *TR* and isoconjugation *QA-Tf2* the vertices P_i have conics *Co*(P_i) inscribed *QA-Tr1* with focus P_i.

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