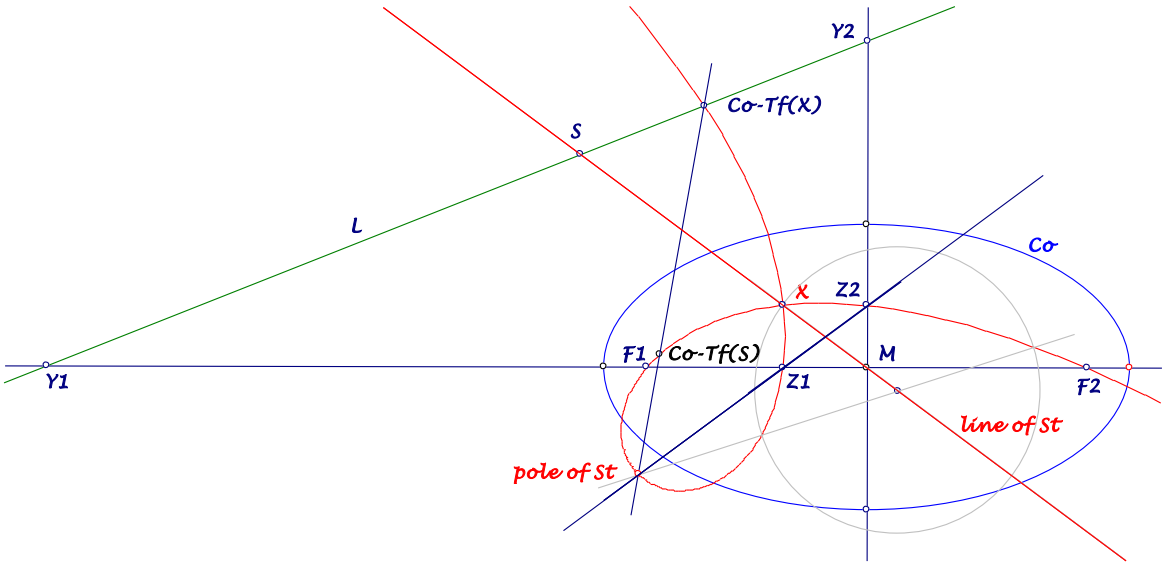
EQF-Note 2018-01-23

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Example for a Conic Transformation

A conic transformation Co-Tf is only determined by a conic Co. Here a simple example shall be researched, mapping a point to the pedal point on its polar wrt a reference conic Co.



Definition: *Co-Tf* is the transformation, which maps a point *P* to the pedal point of *P* on its polar wrt the conic *Co*.

- **Points on the conic *Co* are fixed points of *Co-Tf*.**

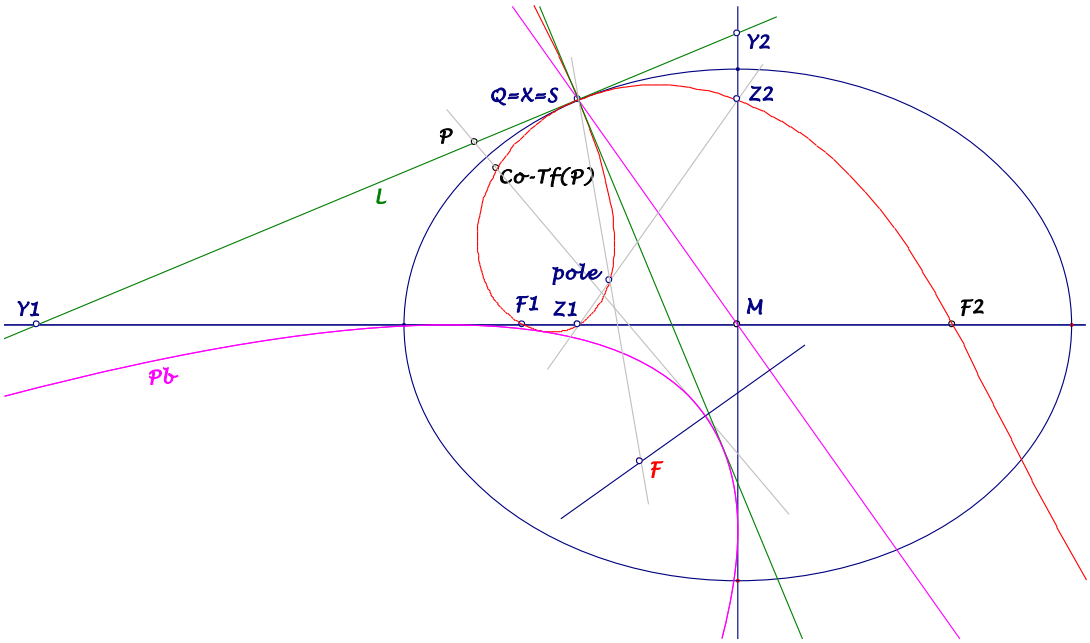
Used points for mapping a line L:
F_i foci of the reference conic *Co*
M center of *Co*,
X pole of *L* wrt *Co*,
S intersection of *L* and *MX*,
Y_i intersections of the *Co*-axes and *L*,
Z_i 4th harmonic points of Y_i
…wrt the intersections of *Co* and its axes.

- **Lines *L* are mapped by *Co-Tf* to strophoids:**
  … fixed point of the strophoid is *X*,
  … line of the strophoid is *MX*,
  … pole of the strophoid is the intersection
  … … $Z_1Z_2 \cap$ *Co-Tf(X).Co-Tf(S)*.

- **The strophoid *Co-Tf(L)* bears**
  … beside pole and fixed point
  … the foci F_i of *Co*,
  … the points Z_i
  … and *Co-Tf(S)* and *Co-Tf(X)*.

- **The strophoid *Co-Tf(L)* degenerates:**
  … For lines through the center of *Co*
  … … to orthogonal hyperbolas
  … … centered in the center of *Co*,
  … … through the foci of *Co*
  … … and the intersections of *L* and *Co*.
  … For lines parallel to the main axis of *Co*
  … … to circles through the foci and the *Co*-pole of *L*.
  … For lines orthogonal to the main axis of Co
  … … to circles with diameter *XU*
  …. … … and *U* 4th harmonic point of *X* wrt the foci.

- **The strophoid *Co-Tf(L)* of a tangent at *Co* in *Q* =*X* = *S*:**
  … The lines *P.Co-Tf(P)* envelope a parabola *Pb*
  … tangent to the axes of *Co*
  … … and to the tangent and normal in *Q*,
  … … with directrix *MQ*
  … … and focus *F* in the reflection of *Q* in the pole.



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