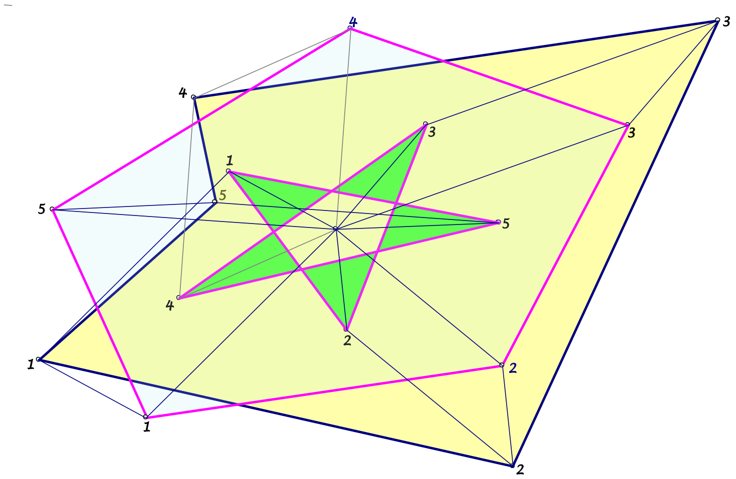
EQF-Note 2018-02-03

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Affinely Regular Components of a 5-Gon

Every 5-gon is uniquely representable as sum of an affinely regular pentagon and an affinely regular pentagram. – This is a special result in the book:
F.Bachmann / E.Schmidt : n-Ecke
Hochschultaschenbuch 471/471a
Bibliographisches Institut AG, Mannheim 1970



In the cited book n-gons are ordered n-tupel of vectors with an addition and multiplication by real numbers, in this way the set of n-gons can be treated as vector space. Here only n-gons with centroid zero are considered!
Researched are transformations, which are polynomials of the basic transformation

$$\zeta : (a_1,a_2, \ldots,a_n) \rightarrow (a_2,a_3, \ldots a_n,a_1).$$

The transformation, which maps a n-gon to the n-gon of the midpoints of the sides, will be

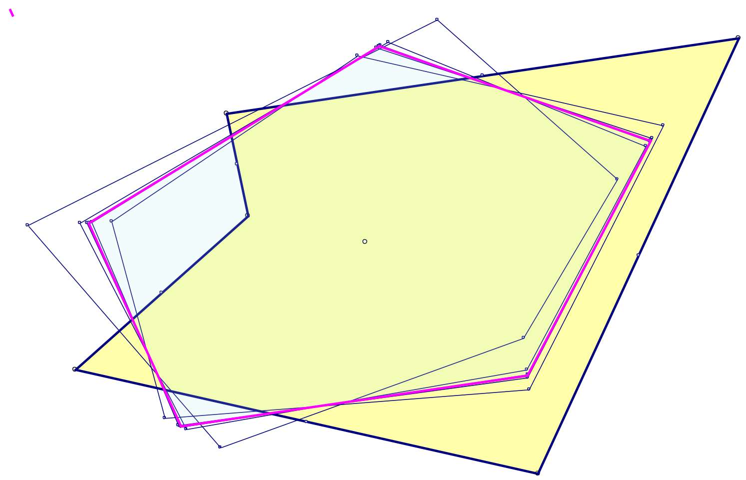
$$\boldsymbol{\kappa_2 = \tfrac{1}{2}(1 + \zeta).}$$

This transformation leads iterated to the n-times counted centroid. But the modified transformation, stretched and shifted indexed,

$$\boldsymbol{\kappa_2^* = -\frac{1}{\cos\pi/5}\,\zeta^2\,\kappa_2}$$

maps affinely regular pentagons to itself. Iterated mapping of $\boldsymbol{\pi_2^*}$ for a 5-gon leads to an affinely regular pentagon, which is the image of the reference 5-gon by the transformation

$$\boldsymbol{\tfrac{2}{5}(1+ \cos2\pi/5\ \zeta + \cos4\pi/5\ \zeta^2 + \cos4\pi/5\ \zeta^3 + \cos2\pi/5\ \zeta^4).}$$



The transformation, which maps a n-gon to the n-gon of the 4th parallelogram points of three consecutive vertices

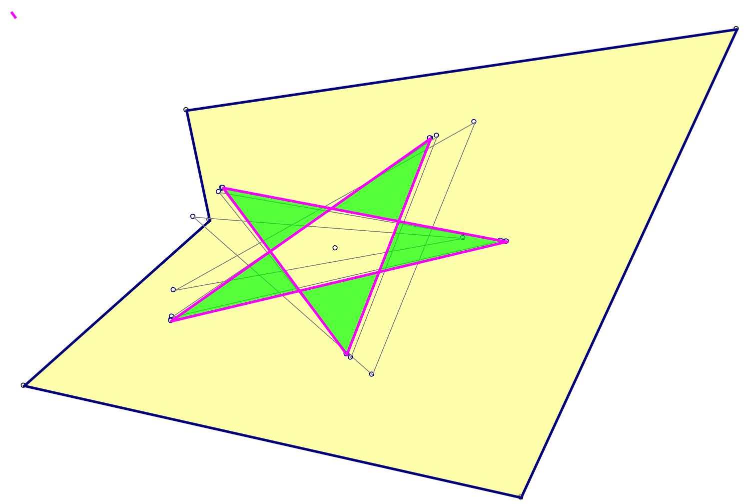
$$\boldsymbol{\pi_3 = 1 - \zeta + \zeta^2}$$

leads iterated used in the modification

$$\boldsymbol{\pi_3^* = -\frac{\sin\pi/10}{\cos\pi/5}\,\zeta^4\,\pi_3}$$

to an affinely regular pentagram, which is the image of the reference 5-gon by the transformation

$$\boldsymbol{\frac{2}{5}(1 + \cos 4\pi/5\ \zeta + \cos 2\pi/5\ \zeta^2 + \cos 2\pi/5\ \zeta^3 + \cos 4\pi/5\ \zeta^4).}$$



These pentagon and pentagram are the components of a 5-gon in the sense of the following theorem (see cited book 12.5):

- **Every 5-gon is uniquely representable as a sum of an affinely regular pentagon and an affinely regular pentagram.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de